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Unpredictability in economic analysis, econometric modeling and forecasting

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ABSTRACT

Unpredictability arises from intrinsic stochastic variation, unexpected instances of outliers, and unanticipated extrinsic shifts of distributions. We analyze their properties, relationships, and different effects on the three arenas in the title, which suggests considering three associated information sets. The implications of unanticipated shifts for forecasting, economic analyses of efficient markets, conditional expectations, and inter-temporal derivations are described. The potential success of general-to-specific model selection in tackling location shifts by impulse-indicator saturation is contrasted with the major difficulties confronting forecasting.

1. Introduction

Unpredictability has been formalized as intrinsic stochastic variation in a known distribution, where conditioning on available information does not alter the outcome from the unconditional distribution, as in the well-known prediction decomposition, or sequential factorization, of a density: see Doob (1953). Such variation can be attributed (*inter alia*) to chance distribution sampling, 'random errors', incomplete information, or in economics, many small changes in the choices by individual agents. A variable that is intrinsically unpredictable cannot be modeled or forecast better than its unconditional distribution.

However, the converse does not hold: a variable that is not intrinsically unpredictable may still be essentially unpredictable because of two additional aspects of unpredictability. The first concerns independent draws from fat-tailed or heavy-tailed distributions, which leads to a notion we call 'instance unpredictability'. Here the distribution of a variable that is not intrinsically unpredictable is known, as are all conditional and unconditional probabilities, but there is a non-negligible probability of a very discrepant outcome. While that probability is known, it is not known on which draw the discrepant outcome will occur, nor its magnitude, leading to a 'Black Swan' as in Taleb (2007), with potentially large costs when that occurs: see Barro (2009). The third aspect we call 'extrinsic unpredictability', which derives from unanticipated shifts of the distribution itself at unanticipated times, of which location shifts (changes in the means of distributions) are usually the most pernicious. Intrinsic and instance unpredictability are close to 'known unknowns' in that the probabilities of various outcomes can be correctly pre-calculated, as in rolling dice, whereas extrinsic unpredictability is more like 'unknown unknowns' in that the conditional and unconditional probabilities of outcomes cannot be accurately calculated in advance as in the first quote of Clements and Hendry (1998). The recent financial crisis and ensuing deep recession have brought both instance and extrinsic unpredictability into more salient focus: see Taleb (2009), and Soros (2008, 2010).

These three aspects of unpredictability suggest that different information sets might explain at least a part of their otherwise unaccounted variation. This is well established both theoretically and empirically for intrinsic unpredictability, where 'regular' explanatory variables are sought. Empirically, population distributions are never known, so even to calculate the probabilities for instance unpredictability, it will always be necessary to estimate the distributional form from available evidence, albeit few 'tail draws' will occur from which to do so. New aspects of distributions have to be estimated when extrinsic unpredictability occurs.

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Consequently, each type of unpredictability has substantively different implications for economic analyses, econometric modeling, and economic forecasting. Specifically, inter-temporal economic theory, forecasting, and policy analyses could go awry facing extrinsic unpredictability, yet *ex post*, the outcomes that eventuated are susceptible to being modeled. We briefly discuss the possible role of impulse-indicator saturation for detecting and removing in-sample location shifts. The availability of such tools highlights the contrast between the possibilities of modeling extrinsic unpredictability *ex post* against the difficulties confronting successful *ex ante* forecasting, where one must forecast outliers or shifts, which are demanding tasks. However, transformations of structural models that make them robust after shifts, mitigating systematic forecast failure, are feasible.

The structure of the paper is as follows. Section 2 considers intrinsic unpredictability in Section 2.1; instance unpredictability in Section 2.2; and extrinsic unpredictability in Section 2.3. Theoretical implications are drawn in Section 3, with the relationships between intrinsic, instance and extrinsic unpredictability in Section 3.1, and the impact of reduced information in Section 3.2. The possibility of three distinct information sets associated respectively with 'normal causality', the timing of outliers, and the occurrence of distributional shifts is discussed in Section 3.3. The difficulties both economists and economic agents confront facing unanticipated breaks are analyzed in Section 3.4. Section 4 investigates some consequences for empirical applications. The fundamental separation between modeling and forecasting from instance and extrinsic unpredictability – but not intrinsic unpredictability – is discussed in Section 4.1. Then Section 4.2 considers the relationships between the three aspects of unpredictability for model selection in processes with unanticipated breaks, leading to a reconsideration of the role of congruent modeling for forecasting in Section 4.3. These analyses are illustrated in Section 4.4 by an empirical application of robust forecasting. Section 5 concludes.

2. Unpredictability

We now consider the three distinct sources of unpredictability. Were it the case that the data generation process (DGP) changed unexpectedly at almost every data point, then reliable inferences would be rendered essentially impossible. Fortunately, the various sources of unpredictability are less extreme than this, so inference remains possible subject to the *caveats* discussed in the following.

2.1. Intrinsic unpredictability

Definition 1. A non-degenerate n-dimensional vector random variable $\boldsymbol{\epsilon}_t$ is an *intrinsically* unpredictable process with respect to an information set $\mathcal{I}_{t-1}$, which always includes the sigma-field generated by the past of $\boldsymbol{\epsilon}_t$, denoted $\sigma[\mathbf{E}_{t-1}]$, over a time period $\mathcal{T}$ if the conditional distribution $\mathsf{D}_{\epsilon_t}(\boldsymbol{\epsilon}_t|\mathcal{I}_{t-1})$ equals the unconditional distribution $\mathsf{D}_{\epsilon_t}(\boldsymbol{\epsilon}_t)$:

$$\mathsf{D}_{\epsilon_t}(\boldsymbol{\epsilon}_t \mid \mathcal{I}_{t-1}) = \mathsf{D}_{\epsilon_t}(\boldsymbol{\epsilon}_t) \quad \forall t \in \mathcal{T}. \tag{1}$$

Intrinsic unpredictability is so-called as it is an intrinsic property of $\boldsymbol{\epsilon}_t$ in relation to $\mathcal{I}_{t-1}$, not dependent on knowledge about $\mathsf{D}_{\epsilon_t}(\cdot)$, so is tantamount to independence between $\boldsymbol{\epsilon}_t$ and $\mathcal{I}_{t-1}$. When $\mathcal{I}_{t-1} = \sigma[\mathbf{X}_{t-1}]$ (say) is the 'universal' information set, (1) clarifies why $\boldsymbol{\epsilon}_t$ is *intrinsically* unpredictable. Intrinsic unpredictability applies equally to explaining the past (i.e., modeling $\{\boldsymbol{\epsilon}_t, t = 1, \ldots, T\}$) and forecasting the future from T (i.e., of $\{\boldsymbol{\epsilon}_t, t = T+1, \ldots, T+h\}$): the best that can be achieved in both settings is the unconditional distribution, and $\mathcal{I}_{t-1}$ is of no help in reducing either uncertainty.

Expectations formed at time t using a distribution f_t are denoted $\mathsf{E}_{\mathsf{f}_t}[\cdot]$, and the variance is denoted $\mathsf{V}_{\mathsf{f}_t}[\cdot]$ for each point in $\mathcal{T}$.

Theorem 1. *When the relevant moments exist, intrinsic unpredictability in distribution entails unpredictability in mean and variance:*

$$\mathsf{E}_{\mathsf{f}_t}[\boldsymbol{\epsilon}_t \mid \mathcal{I}_{t-1}] = \mathsf{E}_{\mathsf{f}_t}[\boldsymbol{\epsilon}_t] \quad \textit{and} \quad \mathsf{V}_{\mathsf{f}_t}[\boldsymbol{\epsilon}_t \mid \mathcal{I}_{t-1}] = \mathsf{V}_{\mathsf{f}_t}[\boldsymbol{\epsilon}_t]. \tag{2}$$

However, neither the former nor the latter alone need imply the other. As a well known example, $\boldsymbol{\epsilon}_t \sim \mathsf{IN}_n[\boldsymbol{\mu}, \boldsymbol{\Omega}_\epsilon]$, denoting an independently distributed Gaussian variable with expected value $\mathsf{E}_{\mathsf{f}_t}[\boldsymbol{\epsilon}_t] = \boldsymbol{\mu}$ and variance $\mathsf{V}_{\mathsf{f}_t}[\boldsymbol{\epsilon}_t] = \boldsymbol{\Omega}_\epsilon$, is an intrinsically unpredictable process.

Intrinsic unpredictability is only invariant under non-singular contemporaneous transformations, as inter-temporal transforms must affect (1), implying that no unique measure of forecast accuracy exists: see e.g., Leitch and Tanner (1991), Clements and Hendry (2005), and Granger and Pesaran (2000a,b). Thus, predictability requires combinations with $\mathcal{I}_{t-1}$, as in, for example

$$\mathbf{y}_t = \boldsymbol{\psi}_t(\mathbf{X}_{t-1}) + \boldsymbol{\epsilon}_t \quad \text{where } \boldsymbol{\epsilon}_t \sim \mathsf{ID}_n[\mathbf{0}, \boldsymbol{\Omega}_\epsilon] \tag{3}$$

so $\mathbf{y}_t$ depends on both the information set and the innovation component. Then

$$\mathsf{D}_{\mathsf{y}_t}(\mathbf{y}_t \mid \mathcal{I}_{t-1}) \neq \mathsf{D}_{\mathsf{y}_t}(\mathbf{y}_t) \quad \forall t \in \mathcal{T}. \tag{4}$$

In (3), $\mathbf{y}_t$ is predictable in mean even if $\boldsymbol{\epsilon}_t$ is unpredictable as

$$\mathsf{E}_{\mathsf{D}_{\mathsf{y}_t}}[\mathbf{y}_t \mid \mathcal{I}_{t-1}] = \boldsymbol{\psi}_t(\mathbf{X}_{t-1}) \neq \mathsf{E}_{\mathsf{D}_{\mathsf{y}_t}}[\mathbf{y}_t],$$

in general. Since

$$\mathsf{V}_{\mathsf{D}_{\mathsf{y}_t}}[\mathbf{y}_t \mid \mathcal{I}_{t-1}] < \mathsf{V}_{\mathsf{D}_{\mathsf{y}_t}}[\mathbf{y}_t] \quad \text{when } \mathsf{V}_t\left[\boldsymbol{\psi}_t(\mathbf{X}_{t-1})\right] \neq \mathbf{0} \tag{5}$$

predictability ensures a variance reduction, consistent with its nomenclature, since unpredictability entails equality in (5), and the 'smaller' the conditional variance matrix, the less uncertain is the prediction of $\mathbf{y}_t$ from $\mathcal{I}_{t-1}$.

2.2. Instance unpredictability

Definition 2. The vector random variable $\boldsymbol{\epsilon}_t$ is an *instance* unpredictable process over a time period $\mathcal{T}$ if there is a non-negligible probability of 'extreme draws' of unknown magnitudes and timings.

As Taleb (2007, 2009) has stressed, rare large-magnitude events, or 'Black Swans', do unexpectedly occur. One formulation of that insight is to interpret 'Black Swans' as highly discrepant draws from fat-tailed distributions, where there is a constant, but small, probability of such an occurrence each period. Both the timings and the magnitudes of the discrepant events are then unpredictable, even when the form of the distribution is known and constant over time. When a large outcome materializes unexpectedly at time τ, say, substantial costs or benefits can result, sometimes both, but for different groups of agents. Barro (2009) estimates very high costs from 'consumption disasters', finding 84 events over approximately the last 150 years with falls of more than 10% per capita, cumulating to a total duration of almost 300 'bad' years across his sample of 21 countries, mainly due to wars. However, there is a marked reduction in their frequency after World War II.

Recent research on many financial variables has revealed a proliferation of 'jumps', as measured by bipower variation: see e.g., Barndorff-Nielsen and Shephard (2004, 2006). These seem to be *ex ante* instance unpredictable events, superimposed on the underlying Ornstein–Uhlenbeck processes. To mitigate possibly large costs from financial outliers, Taleb (2009) argues for more robust systems that avoid dependence on the non-occurrence of very large draws: for example, systems with more nodes and less interdependence.