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On the optimality of funding and hiring/firing according to stochastic demand: The role of growth and shutdown options

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ABSTRACT

We examine the firm's investment and hiring/firing policy under stochastic demand with potential reversibility. We evaluate in particular the values of both investment and hiring/firing growth and shutdown options not only for the standard Cobb–Douglas production function but also when taking account of the natural upper bound on the output due to the demand level. For this latter purpose, we use results about average of options provided in Shackleton and Wojakowski (2007). As a by-product, we extend the approach of Tserlukevich (2008) by introducing the employment level to analyze in particular the optimality of the financial structure and leverage. Our approach allows us to get a quasi-explicit solution of the optimal firm's value that can be deeply analyzed. Such results can potentially explain the interest for flexible contractual arrangements with capital and labor firm's structure.

1. Introduction

The literature on investment theory and cost of capital has been initiated by the seminal paper of Modigliani and Miller (1958). Further extensions have examined its robustness with respect to various market frictions including for example transaction costs. Real frictions, taking account of imperfections, can explain why Modigliani and Miller assumptions can be violated. For instance, there exists an uncertainty about the level of future cash-flows. Additionally, investment is often partially irreversible.[1]

As proved by Pindyck (1988), the investor can benefit from the option to wait before investing. Usually, the option to invest is exercised as soon as the expected discounted cash-flows are higher than the sunk investment expenditures. McDonald and Siegel (1986) have illustrated this case when the value of the investment project evolves as a geometric Brownian motion and only one output is produced. In that case, the optimal investment strategy is a trigger one: the option to invest is exercised at the first time the value of the investment project exceeds a critical threshold (see Dixit and Pyndick, 1994).

As mentioned by Décamps et al. (2006), research on investment under uncertainty has also emphasized the key role of entry and exit decisions (Dixit, 1989), the flexibility of incremental capacity choice (Kandel and Pearson, 2002; Pindyck, 1988), the shutdown options (McDonald and Siegel, 1986), the costly reversibility (Abel and Eberly, 1994, 1996; Laughton and Jacoby, 1993), and finally, the sequential nature of investment decision (Bar-Ilan and Strange, 1998; Majd and Pindyck, 1987). Therefore, both financing and investing decisions must be analyzed according to such real options, as illustrated by Quigg (1993, 1995), Schwartz (1988, 1997), Trigeorgis (1996) and Abel and Eberly (1994, 1999). Extending previous results by Abel and Eberly (1996), Tserlukevich (2008) introduces a model that can potentially explain some empirical financing patterns. Investment irreversibility can be either complete or partial, with or without fixed investment costs. One single or multiple growth options can be available. When the market demand follows a geometric Brownian motion, the leverage ratio can be constant or can be time-varying according to the presence of the growth option. Following Capozza and Li (1994) and Bar-Ilan and Strange (1999) but introducing an abandonment option, Wong (2010) examines how changes in irreversibility of investment affect the timing and intensity of lumpy investment.

In this paper, contrary to Abel and Eberly (1996) and Tserlukevich (2008), we introduce another important factor namely the employment level. As illustrated by Faria et al. (2010), there exists a strong relation between entrepreneurship and employment level.[2] More generally, the employment level of a given firm and its potential flexibility[3] have a big impact on its performances that we illustrate on the global firm

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[1] As emphasized by Arrow and Fisher (1974), the consequences of irreversibility can be severe when dealing with uncertainty.

[2] See also Andersen and Svarer (2007) who study the flexi-security properties; Blasco and Pertold-Gebicka (2013) who investigate in particular the link between firm's performance and employment.

[3] See He and Pindyck (1992) for the role of flexible production capacity.

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value, the debt value and the leverage ratio. Taking account of this important feature, we analyze both the investment/divestment and hiring/firing decisions in a stochastic demand framework with potential reversibility. The introduction of such potential hiring/firing provides a second extension of the results by Abel and Eberly (1996) and Tserlukevich (2008). The cash flow of the firm is stochastic over time. It is assumed to be increasing with respect to both the investment and the employment levels. The firm has two types of options: a growth option if the market demand rises sufficiently; and an option to reduce both its investment and employment levels if the market demand decreases substantially. In this latter case, we show that the optimal strategy is the shutdown option, which means that the project is abandoned. Finally, we consider the case of upper bounded output quantity, which also extends previous works. For all the cases, we prove that the optimal decisions are triggered, which means that they are based on whether or not the demand level reaches given thresholds.

The paper is organized as follows. Section 2 deals with the optimal investment problem with and without the growth and shutdown options for the standard Cobb–Douglas production function. We introduce the employment level to analyze in particular its impact on the optimality of the financial structure and leverage. Section 3 deals with the investment and hiring/firing growth and shutdown options when taking account of the natural upper bound on the output due to the demand level. Our approach allows us to get quasi-explicit solutions of the optimal firm's value and the growth and shutdown options that are further analyzed. Most of the proofs are gathered in appendices.

2. Optimal investment and hiring/firing policy with one option

In this first section, the firm has a single option to modify its initial investment and hiring/firing strategy. We investigate three main cases:

- The first one corresponds to potential investment increase and to hiring. The firm's manager searches to determine the optimal time to invest and to hire and the corresponding new levels of investment and employment. He benefits from the growth option, as soon as the demand for the product sold by the firm is sufficiently high;
- The second one is linked to the prospect of divesting and fire. Taking account of potential reduced demand, the firm's manager tries to optimally divest and to fire. He uses a shutdown option, as soon as the demand for the product sold by the firm becomes significantly low;
- Finally, we consider both previous cases. Depending on demand fluctuations, the firm's manager will use the growth option or the shutdown option. Both are included in the manager's decision process and the use of one instead of the other depends on the random demand.

Note that each of these three operations is irreversible; which means that all these options can be exercised only once.

Demand for the product sold by the firm corresponds here to the selling price of one unit of the output. It is modeled through a Geometric Brownian Motion (GBM) S satisfying the following stochastic differential equation (SDE):

$$dS_t = S_t[\mu dt + \sigma dW_t],$$

where μ is a constant drift, σ denotes the constant volatility and W is a standard Brownian motion.

At initial time, the levels of the firm's investment and employment are respectively equal to K_0 and L_0. The production function is assumed to be a Cobb–Douglas function. It means that the profit of the firm is given by:

$$\pi(S_t, K_0, L_0) = S_t K_0^{\alpha} L_0^{\beta}, \text{ with } 0<\alpha,\beta<1.$$

There exists a tax τ on interest income, which implies a tax payment $(1-\tau)$ that reduces the riskless interest rate r. We assume also that the time horizon is infinite.

From previous assumptions, we deduce the net present value without any growth or shutdown option (see Tserlukevich, 2008).

Proposition 1. *At any time t, the net present value V_t satisfies:*

$$V_t(S_t, K_0, L_0) = \mathbb{E}_t\left[\int_t^{\infty} e^{-r(1-\tau)(s-t)} S_s K_0^{\alpha} L_0^{\beta} ds\right] = \frac{K_0^{\alpha} L_0^{\beta}(1-\tau)}{r(1-\tau)-\mu} S_t. \tag{1}$$

The value of the equity E_t is equal to the net present value minus the value of the debt D_t. Thus we deduce:

$$E_t(S_t, K_0, L_0) = V_t(S_t, K_0, L_0) - D_t.$$

2.1. Investment and hiring (growth option)

The firm's manager can increase his investment from K_0 to $K_{1,}^{+}$ and the employment level from L_0 to $L_{1,}^{+}$ at the same random time T_1^{+}. He searches for the maximum value of the equity $E_{I,t}(S_t,K_0,L_0)$ using the growth option. A project once installed incurs no operating costs. However, when for example investing and hiring (also for the divesting and firing case), the manager entails sunk capital and employment costs denoted by $C^{+}(K_0,K_1^{+},L_0,L_{1,}^{+})$.

Therefore, the optimization problem is given by: for any current time t before T_1^{+},

$$\begin{aligned} E_{I,t}^{+}(S_t, K_0, L_0) = & \max_{T_1^{+}} (1-\tau)\mathbb{E}_t\left[\int_t^{T_1^{+}} e^{-r(1-\tau)(s-t)} K_0^{\alpha} L_0^{\beta} S_s ds\right] \\ & + \max_{K_1^{+}, L_1^{+}} (1-\tau)\mathbb{E}_t\left[\int_{T_1^{+}}^{\infty} e^{-r(1-\tau)(s-t)} K_1^{+\alpha} L_1^{+\beta} S_s ds\right] \\ & - \mathbb{E}_t\left[e^{-r(1-\tau)(T_1^{+}-t)} C\left(K_0, K_{1,}^{+} L_0, L_{1,}^{+}\right)\right] - D_t. \end{aligned} \tag{2}$$

The residual value of the equity $E_{I,t}^{+}(S_t K_0, L_0)$ is the sum of the net present value at time t corresponding to initial capital K_0 and employment level L_0 net of debt, that is

$$\max_{T_1^{+}} (1-\tau)\mathbb{E}_t\left[\int_t^{T_1^{+}} e^{-r(1-\tau)(s-t)} K_0^{\alpha} L_0^{\beta} S_s ds\right] - D_t \tag{3}$$

and a part of the growth option value equal to:

$$\max_{K_1^{+}, L_1^{+}} (1-\tau)\mathbb{E}_t\left[\int_{T_1^{+}}^{\infty} e^{-r(1-\tau)(s-t)} K_1^{+\alpha} L_1^{+\beta} S_s ds\right] - \mathbb{E}_t\left[e^{-r(1-\tau)(T_1^{+}-t)} C\left(K_0, K_1^{+}, L_0, L_{1,}^{+}\right)\right]. \tag{4}$$

We consider the cost associated with the additional investment and labor levels. For the investing/hiring case, both costs are proportional to respectively $(K_1^{+} - K_0)$ and $(L_1^{+} - L_0)$ with proportion coefficients respectively equal to $p^{+} > 1$ and $q^{+} > 1$:

$$C\left(K_0, K_1^{+}, L_0, L_1^{+}\right) = p^{+}\left(K_1^{+} - K_0\right) + q^{+}\left(L_1^{+} - L_0\right). \tag{5}$$

The optimal time T_1^{+} corresponds to the first time at which the demand S reaches the barrier S_{max}^{*}.