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Statistical microeconomics

Belal E. Baaquie *

Department of Physics, National University of Singapore, Singapore
Risk Management Institute, National University of Singapore, Singapore

HIGHLIGHTS

- A formulation based on statistical mechanics is proposed for microeconomics.
- A potential is introduced to explain the interplay of supply and demand.
- The market equilibrium prices are defined by the minima of the potential.
- A Lagrangian is defined to model the dynamics of commodities.
- A brief discussion is given on the perturbation expansion for this model of microeconomics.

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ABSTRACT

A statistical generalization is made of microeconomics in the spirit of going from classical to statistical mechanics. The price and quantity of every commodity[1] traded in the market, at each instant of time, is considered to be an *independent random variable*: all prices and quantities are considered to be stochastic processes, with the observed market prices being a random sample of the stochastic prices. The dynamics of market prices is determined by an *action functional* and, for concreteness, a specific model is proposed. The model can be calibrated from the unequal time correlation of the market commodity prices. A perturbation expansion for the correlation functions is defined in powers of the inverse of the total budget of the aggregate consumer and the propagator for the market prices is evaluated.

1. Introduction

The synthesis of economics and physics has given rise to the new subject of Econophysics [1]. Most of the studies in econophysics have been focused on the financial markets [2,3] and on financial instruments and their derivatives [4,5].

Microeconomics is one of the pillars of modern economic theory and studies the interaction of consumers and producers of commodities [6–8]. There is increasing research in the application of statistical physics to economics [9–12] and this paper is a continuation of such studies.

Let quantity $\mathbf{q} = (q_1, q_2, \ldots, q_N)$, where $q_i > 0$, be the quantity of a commodity labeled by i, with $i = 1, 2, \ldots, N$; it can be kilograms of wheat or the number of automobiles. The commodity price vector is $\mathbf{p} = (p_1, p_2, \ldots, p_N)$, where $p_i > 0$ is the price of a unit of the commodity; it can be dollars/kilograms or dollars/per automobile.

One of the fundamental problems of microeconomics is to determine the dependence of quantities $\mathbf{q}$ on the purchased market prices $\mathbf{p}$.

* Correspondence to: Department of Physics, National University of Singapore, Singapore. Tel.: +65 68742963.
E-mail addresses: phybeb@nus.edu.sg, belalbaaquie@gmail.com.

1 The term commodities is used for goods and services.

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In most studies of microeconomics, at a given instant, the quantity and price of a commodity are taken to be a determinate quantity. Microeconomics studies the (deterministic) equilibrium value of the quantities and prices of commodities as well their time evolution. A statistical generalization is made in microeconomics by considering quantities $q_i(t)$ and price $p_i(t)$ to be independent random variables for each instant of time, namely *stochastic variables*.

A possible reason for prices to be random is that, similar to the price of equities, the prices of commodities incorporate all the market information and result in the traded prices. In the absence of new information, any departures from the traded prices, hence, should be indeterminate, random and uncertain. Furthermore, market prices are not in equilibrium, but rather have a (random) evolution in time t that can have an overall drift reflecting market sentiment. Market prices may not contain all the market information and the source of randomness of market prices may have other explanations such as due to the existence of 'sticky' prices [13].

In statistical microeconomics, the supply $\mathcal{S}[\mathbf{p}]$ and demand $\mathcal{D}[\mathbf{p}]$ of commodities at market prices $\mathbf{p}$ is the starting point for analyzing the behavior of the producers and consumers of commodities. The competing tendency of demand and supply, namely demand increases when prices fall whereas supply increases when prices rise is reflected in the traded prices. In fact, in most microeconomics texts, the market commodity price is taken to be the value for which supply is equal to demand.

Supply and demand are inseparable, with one determining the other and vice versa. The view taken in this paper is that supply and demand are two facets of the same entity, namely a microeconomic *potential function* $\mathcal{V}[\mathbf{p}]$. Using the analogy from mechanics, a potential function $\mathcal{V}[\mathbf{p}]$ is postulated that *combines* supply and demand into a single entity and embodies the competing effects of both supply and demand. As will be discussed later, both the supply and demand functions are dimensionless and hence can be consistently added together. The potential is chosen to be the sum of supply and demand, namely

$$\mathcal{V}[\mathbf{p}] = \mathcal{D}[\mathbf{p}] + \mathcal{S}[\mathbf{p}]. \tag{1}$$

The potential function $\mathcal{V}[\mathbf{p}]$, similar to mechanics, drives the evolution of market prices. For the special case when the prices are constant (time independent) – given by the constant prices $\mathbf{p}_0 = (p_{01}, p_{02}, \ldots, p_{0N})$ – the prices *minimize the value* of the potential; namely that $\mathcal{V}[\mathbf{p}_0]$ is a minimum of $\mathcal{V}[\mathbf{p}]$. The framework of statistical microeconomics, stationary prices are determined by the minimization of the microeconomic potential, which replaces the standard microeconomic procedure of setting supply equal to demand [6].

The full dynamics of market prices is determined by assigning a *joint probability distribution* for all possible evolutions of the stochastic market prices. In analogy with quantum mechanics and classical statistical mechanics, it is *postulated* that the probability of the stochastic evolution of market prices is proportional to the Boltzmann distribution, namely

$$\text{Joint probability distribution} \propto \exp\{-\mathcal{A}[\mathbf{p}]\} \tag{2}$$

where the action functional $\mathcal{A}[\mathbf{p}]$ determines the likelihood of the evolution of all the different values taken by all the prices.

In analogy with mechanics, the action functional is taken to be the sum of the potential term $\mathcal{V}[p]$ with a *kinetic term* $\mathcal{T}$, namely

$$\mathcal{A}[\mathbf{p}] = \int_{-\infty}^{+\infty} \mathrm{d}t\,\mathcal{L}(t) = \int_{-\infty}^{+\infty} \mathrm{d}t\Big(\mathcal{T}[\mathbf{p}(t)] + \mathcal{V}[\mathbf{p}(t)]\Big) \tag{3}$$

with the Lagrangian given by

$$\mathcal{L}(t) = \mathcal{T}[\mathbf{p}(t)] + \mathcal{V}[\mathbf{p}(t)]. \tag{4}$$

The kinetic term $\mathcal{T}[\mathbf{p}(t)]$ contain the time derivatives of the prices and together with the potential function, determines the time dependence of the stochastic prices; in particular, $\exp\{-\mathcal{A}[\mathbf{p}]\}$ determines the likelihood of the different random trajectories of the random prices.

Note that for all values of the prices $\mathcal{A}[\mathbf{p}] > 0$; the minimum value of $\mathcal{A}[\mathbf{p}]$ has no significance, with the only requirement being that the minimum value is finite; by adding a constant, the minimum value of $\mathcal{A}[\mathbf{p}]$ can always be taken to be zero.

To examine the specific characteristics of the statistical formulation of microeconomics, the total budget m of a typical aggregate consumer is introduced as an expansion parameter. In particular, the correlation of the prices is studied as a perturbative expansion in a power series in $1/m$. The perturbation expansion shows that the average prices of the model, to leading order in $1/m$, are equal to the time independent stationary prices $\mathbf{p}_0 = (p_{01}, p_{02}, \ldots, p_{0N})$ that minimize the potential. The series expansion of the unequal time price correlator – in a power series in $1/m$ – can be generated using the technique of Gaussian path integration.

The model can be calibrated by comparing the model's unequal time correlation function with the empirical correlation of market commodity prices.

2. The utility function

The utility function $\mathcal{U}$ is one of the fundamental concepts in microeconomics and depends on the quantity of consumption vector $\mathbf{q} = (q_1, q_2, \ldots, q_N)$ of commodities, that is, $\mathcal{U} = \mathcal{U}[\mathbf{q}]$. The utility function is a *dimensionless real number* that quantifies the utility of a commodity to the consumer, which is necessarily subjective. In all discussions in this paper, the