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How much inequality in income is fair? A microeconomic game theoretic perspective

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HIGHLIGHTS

- Fairest inequality of income is a lognormal distribution under ideal conditions.
- Ideal free market can "discover" the fairest distribution in practice, if allowed.
- Scandinavia is close to ideal inequality for the bottom 99% of the population.
- US was closer to ideal inequality in 1945–75 than it is now for the bottom 90%.
- Deep connection between potential game theory and statistical mechanics via entropy.

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ABSTRACT

The increasing inequality in income and wealth in recent years, and the associated excessive pay packages of CEOs in the US and elsewhere, is of growing concern among policy makers as well as the common person. However, there seems to be no satisfactory answer, in conventional economic theories and models, to the fundamental questions of what kind of income distribution we ought to see, at least under ideal conditions, in a free market environment, and whether this distribution is fair. We propose a novel microeconomic game theoretic framework that addresses these questions and proves that the lognormal distribution is the fairest inequality of pay in an organization comprising of homogeneous agents, under ideal free market conditions at equilibrium. We also show that for a population of two different classes of agents, the equilibrium distribution is a combination of two different lognormal distributions where one of them, corresponding to the top ∼3–5% of the population, can be misidentified as a Pareto distribution. We compare our predictions with empirical data on global income inequality trends provided by Piketty and others. Our analysis suggests that the Scandinavian countries, and to a lesser extent Switzerland, Netherlands and Australia, have managed to get close to the ideal distribution for the bottom ∼99% of the population, while the US and UK remain less fair at the other extreme. Other European countries such as France and Germany, and Japan and Canada, are in the middle. Our theory also shows the deep and direct connection between potential game theory and statistical mechanics through entropy, which we identify as a measure of fairness in a distribution. This leads us to propose the *fair market hypothesis*, that the self-organizing dynamics of the ideal free market, i.e., Adam Smith's "invisible hand", not only promotes efficiency but also maximizes fairness under the given constraints.

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1. Introduction

In recent years, there has been growing concern over the widening inequality in income and wealth distributions in the US and elsewhere [1–5]. The statistics are troubling — for instance, as of 2012, the top 1% of households in the US owned 41.8% of all privately held wealth [6], and it had risen from a low of about 20% in 1976 [7].

An important source of the wealth inequality is a similar trend in the income and pay (or wage) distributions. Income remains highly concentrated, with the top 1% of income earners received 17.9% of all income in 2012 in the US, and that is up from 12.8% in 1982 [7,8]. A related trend of equally great concern is the excessive pay packages for CEOs which are reflected in the extraordinarily high CEO pay ratios, particularly in the US [9,10]. There is much discussion both in academic literature and popular press about what all these mean, what the consequences are, and what can or should be done about it [1–5,11–16].

Obviously, before policy actions, if any, are taken to address these challenges, we need to understand more deeply why and how such inequalities occur. Since different people have *different* abilities and therefore make *different* contributions in a society, naturally, we do expect people to be compensated *unequally*, commensurate with their contributions. Hence, we would expect to see unequal distributions in income and in wealth. So, a certain *level of inequality* is to be expected. But, at the risk of sounding oxymoronic, *what is the fairest level of inequality*? In particular, in an *ideal free market environment*, *what is the level of inequality we ought to see*? This is the question we address in this paper.

While there is extensive empirical literature on income and wealth distributions, and we cite only a sample here [3,6,9,11,12,17–20], there is no satisfactory answer to this question in conventional economic theories and models. Empirical observations are obviously very important, but it would be quite helpful to complement them with a theoretical framework that provides a new useful perspective and analytical insight. From a theoretical perspective, two fundamental questions one would like answered are: *What kind of pay distribution should arise, under ideal conditions, in a free market environment comprising of utility maximizing employees and profit maximizing companies? Is this distribution fair?*

The answers to these questions can serve as a fundamental benchmark against which we can evaluate the distributions seen in real life. In the absence of such a reference framework, the conclusions we reach by relying on empirical observations alone are likely to be incomplete, in an important manner. This benchmark can help us measure and better understand the deviations caused by nonidealities in the real world, and to develop appropriate policy frameworks and incentive structures to try to correct the inequalities. It can give us a quantitative basis for understanding and developing rational tax policies, pay packages for executives, and so on. Our objective in this paper is to develop such a benchmark by proposing a novel microeconomic framework that predicts and explains the emergence of an appropriate pay distribution under ideal free market conditions.

When one explores outside mainstream economics in search of answers to these questions, one finds that there has been much work, in the past decade or so, in the econophysics community to model income and wealth distributions by applying concepts and techniques from statistical mechanics [20–35]. While these models are quite interesting and instructive, they have not, however, bridged the rather wide conceptual gulf that exists between economics and econophysics [36,37], particularly in two crucial areas. One, the typical particle model of agent behavior in econophysics assumes agents to have nearly "zero intelligence", acting at random, with no intent or purpose. This does not sit well with an extensive body of economic literature spanning several decades, where one models, in the ideal case, a perfectly rational agent whose goal is to maximize its utility or profit by acting strategically, not randomly. From the perspective of an economist, it is quite reasonable to ask "How can theories and models based on the collective behavior of purpose-free, random, molecules explain the collective behavior of goal-driven, optimizing, strategizing men and women?"

Another conceptual stumbling block is the role of entropy in economics. In statistical thermodynamics, equilibrium is reached when entropy, which is a measure of randomness or uncertainty, is maximized. So, an economist wonders, why would maximizing randomness or uncertainty be helpful in economic systems? We all know that markets are stable, and function well, when things are orderly, with less uncertainty, not more. This has led to an uneasy relationship with entropy in economics, typically ranging from grudging acceptance to outright rejection, as seen from the remarks of two Nobel Laureates in economics, Amartya Sen and Paul Samuelson. Sen observed [38], while commenting on the Theil Index, "given the association of doom with entropy in the context of thermodynamics, it may take a little time to get used to entropy as a good thing ('How grand, entropy is on the increase!'), but it is clear that Theil's ingenious measure has much to be commended. …But the fact remains that it is an arbitrary formula, and the average of the logarithms of the reciprocals of income shares weighted by income shares is not a measure that is exactly overflowing with intuitive sense. It is, however, interesting that the concept of entropy used in the natural sciences can provide a measure of inequality that is not immediately dismissible, however arbitrary it may be". Similar objections were raised by Samuelson [39]: "As will become apparent, I have limited tolerance for the perpetual attempts to fabricate for economics concepts of 'entropy' imported from the physical sciences or constructed by analogy to Clausius–Boltzmann magnitudes". Thus, we run into major conceptual hurdles in the typical statistical mechanics-based approaches to problems in economics, particularly in the study of income and wealth distributions.

Besides these conceptual challenges, there is also a technical one due to the nature of the datasets in economics. As Ormerod [37] and Perline [40] discuss, one can easily misinterpret data from lognormal distributions, particularly from truncated datasets, as inverse power law or other distributions. Therefore, empirical verification of econophysics models is still in the early stages.