Applied Mathematical Modelling 37 (2013) 3200–3215

Contents lists available at SciVerse ScienceDirect

Applied Mathematical Modelling

journal homepage: www.elsevier.com/locate/apm

A production-repairing inventory model with fuzzy rough coefficients under inflation and time value of money

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ARTICLE INFO

Article history:
Received 9 November 2011
Received in revised form 8 June 2012
Accepted 6 July 2012
Available online 17 August 2012

Keywords:
Dynamic production rate
Trust measure
Fuzzy rough variable
Fuzzy rough expectation

ABSTRACT

In this paper, a production-repairing inventory model in fuzzy rough environment is proposed incorporating inflationary effects where a part of the produced defective units are repaired and sold as fresh units. Here, production and repairing rates are assumed as dynamic control variables. Due to complexity of environment, different costs and coefficients are considered as fuzzy rough type and these are reduced to crisp ones using fuzzy rough expectation. Here production cost is production rate dependent, repairing cost is repairing rate dependent and demand of the item is stock-dependent. Goal of the research work is to find decisions for the decision maker (DM) who likes to maximize the total profit from the above system for a finite time horizon. The model is formulated as an optimal control problem and solved using a gradient based non-linear optimization method. Some particular cases of the general model are derived. The results of the models are illustrated with some numerical examples.

1. Introduction

Different types of uncertainty such as randomness [1], fuzziness [2,3] and roughness [4] are common factors in any real life problem including inventory control. Well established mathematical tools are available to deal with problems involving these uncertainties [2,3,5,1]. But in real life, some problems occur where both fuzziness and roughness exist simultaneously. To overcome these situations normally fuzzy rough variables are used to model the problem. Dubois and Prade [6] introduced the concept of fuzzy rough sets. After that, some researchers [7,8] defined fuzzy rough set as a more general case. Using this approach some researchers modelled different problems where fuzziness and roughness occur simultaneously [9–14]. Liu [15] proposed some definitions and discussed some valuable properties of fuzzy rough variable.

In many cases, it is found that some inventory parameters involve both the fuzzy and rough uncertainties. For example, the inventory related costs – holding cost, set-up cost, production cost, repairing cost, disposal cost, etc. depend on several factors such as bank interest, inflation, labour wages, wear and tear cost, etc. which are uncertain in fuzzy rough sense. To be more specific, inventory holding cost is sometimes represented by a fuzzy number and it depends on the storage amount which may be imprecise and range within an interval due to several factors such as scarcity of storage space, market fluctuation, human estimation/ thought process i.e. it may be represented by a rough set. In the literature, some researchers

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http://dx.doi.org/10.1016/j.apm.2012.07.024

[16,1,17,18] developed and solved inventory models in fuzzy random environment. Very few investigations [9,10] are available for inventory or production-inventory problems with fuzzy rough impreciseness.

Production-repairing system is, now-a-day an important area of inventory studies due to growing environmental concern, environmental regulations in industry and gradually decreasing resources in the world. Beside fabricating the finished product from raw materials, it may be possible to repair the defective units produced from the production. In these cases, recovery of the defective units is economically more attractive than disposal. Furthermore, in the recent past, the growth of environment movement has given the reuse system increasing attention [19]. Initial attempts to address the inventory of repairing items or products dates back to the 1960s, with Schrady [20] being the first to investigate a repair-inventory system. After that, extensive research works have been made to develop real-life recycling models during last two decades[3,21–24]. Some scholars also studied inventory problem in repairing/reworked processes under mixed uncertain environment [10,25].

Defective units may be reworked in the same cycle along with the normal production after some time from the initial commencement of production engaging some additional labour forces and machinery for repair. But all of the defective units can not be considered for reworking. Some of the defective units may be of very poor quality so that it will be expensive to repair those units, which should be avoided for rework. Therefore, a certain percentage of the defective units may be considered for reworking. The demand of the units will be met not only from the produced perfect units but also from the reworked units.

From financial standpoint, an inventory represents a capital investment and must compete with other assets within the firm's limited capital funds. Most of the classical inventory models did not take into account the effects of inflation and time value of money. This has happened mostly because of the belief that inflation and time value of money do not influence the cost and price components (i.e., the inventory policy) to any significant degree. But, during last few decades, due to high inflation and consequent sharp decline in the purchasing power of money in the developing countries like India, Bangladesh etc., the financial situation has been changed and so it is not possible to ignore the effect of inflation and time value of money any further. Misra [26], Chang [27], Sarkar and Moon [28] and Sarkar et al. [29,30] have extended their approaches to different inventory models by considering the time value of money, different inflation rates for the costs, finite replenishment, shortages etc.

After all these studies, some lacunas in the formation of the models and the shortcomings may be summarized as below:

- Most of the earlier models considered constant production/repairing rates and constant production/repairing costs. But in many production systems, production cost is function of production rate [31,32] with imprecise parameters.
- One of the weaknesses of major production-inventory models is the unrealistic assumption that all units produced are of good quality. But production of defective units [33,34] is a natural phenomenon in any production process.
- Normally production inventory models are formulated for infinite time horizon which is not realistic [5]. Due to rapid change of world economy, specially for fashionable/luxarious items, manufacturers very frequently change their product specification with new features, names and packets to attract the customers. As a result lifetime of these products (with respect to demand) in the market is finite[2,32]. Not much attention has been paid on production inventory models in finite time horizon specially for the production repairing system.
- A little study has been made on production repairing model considering selling price, production cost, holding cost, set-up cost, etc. as fuzzy rough parameters [9].
- No investigation on production-repairing inventory system has been made considering dynamic production and repairing rates.

In order to overcome the above mentioned limitations of reworked inventory models, in this paper, a production-repairing inventory model incorporating inflation and time value of money with fuzzy rough coefficients is considered with displayed inventory level dependent demand, dynamic production and repairing rates, production rate dependent production cost and repairing rate dependent repairing cost. Some particular cases of the general model such as models with constant demand having constant/linear production and repairing rates and costs are presented. Numerical examples are provided to illustrate the model.

Rest of the paper is organized as follows. In Section 2, some preliminaries and deductions are presented. In Section 3, assumptions and notations of the proposed inventory model are listed. In Section 4, mathematical formulation of the inventory model is presented. Numerical examples to illustrate the models are provided in Section 5. Some particular cases are presented in Section 6, also. Finally, a brief conclusion is drawn in Section 7. Some mathematical calculations are presented in Appendix.

2. Preliminaries and deductions

2.1. Possibility (Pos), necessity (Nes) measure

Any fuzzy subset $\tilde{a}$ of $\mathcal{R}$ with membership function $\mu_{\tilde{a}}(x) : \mathcal{R} \to [0, 1]$ is called a fuzzy number. Let $\tilde{a}$ and $\tilde{b}$ be two fuzzy quantities with membership functions $\mu_{\tilde{a}}(x)$ and $\mu_{\tilde{b}}(x)$ respectively. Then according to Dubois and Prade [35,36] and Liu and Iwamura [37]