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# Forecasting multivariate time series under present-value model short- and long-run co-movement restrictions

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ABSTRACT

Using a sequence of VAR-based nested multivariate models, we discuss the different layers of restrictions that are imposed on the VAR in levels by present-value models (PVM hereafter) for series that are subject to present-value restrictions. Our focus is novel: we are interested in the short-run restrictions entailed by PVMs (Vahid & Engle, 1993, 1997) and their implications for forecasting.

Using a well-known database, maintained by Robert Shiller, we implement a forecasting competition that imposes different layers of PVM restrictions. Our exhaustive investigation of several different multivariate models reveals that better forecasts can be achieved when restrictions are applied to the unrestricted VAR. Moreover, imposing short-run restrictions produces forecast winners 70% of the time for the target variables of PVMs and 63.33% of the time when all variables in the system are considered.


## 1. Introduction

The use of multivariate time series models in economics and finance has proved fruitful, since they contain key inter-relationships between the variables being modelled. Unfortunately, most of these models have an abundance of free parameters, which poses a problem when they are used for forecasting, since their forecast accuracy measures are usually outperformed by those of more parsimonious alternatives. One way to cope with this forecasting problem is to impose restrictions, thus reducing the number of free parameters in the forecasting models. This is often done for small-dimension vector autoregressive (VAR) models by testing and imposing long-run relationships among the series being modelled when they individually trend and jointly co-trend over time (see Engle & Granger, 1987; Johansen, 1988). One can also impose further commonalities in their short-run dynamics, e.g., impose common cyclical feature restrictions (see Engle & Kozicki, 1993; Vahid & Engle, 1993).

The extensive work on cointegration has indeed shown that considering and imposing long-run relationships leads to forecasting gains compared to the model in first differences (see also Clements & Hendry, 1998, or Hoffman & Rasche, 1996, *inter alia*). However, only a handful of papers (e.g., Anderson & Vahid, 2011; Issler & Vahid, 2001; Vahid & Issler, 2002) have investigated whether including additional short-run co-movement restrictions generates better forecasts. Moreover, Athanasopoulos, Guillen, Issler, and Vahid (2011) only recently compared the relative im-

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portance of these two types of restrictions using simulations and real data, and showed that the existing short-run restrictions have a greater potential to improve the forecast accuracy than cointegration restrictions.

Both short- and long-run restrictions are implied by the present-value model (PV model or PVM, hereafter) introduced by Campbell and Shiller (1987) and studied here. However, most papers have focused on the presence of cointegration between the levels of two variables (labeled $Y_t$ and $y_t$ in this paper), a condition that is necessary for the validity of a present-value model linking them.[1] Hence, it is often overlooked that *another* necessary condition for the PVM to hold is that the forecast error implied by the PV model be orthogonal to the past. We refer to the studies by Baillie (1989) and Hansen and Sargent (1981, 1993) for initial work on rational expectations linked to PVMs, and those by Johansen (2000) and Johansen and Swensen (1999, 2004, 2011) for a recent fresh look at the subject.

Indeed, PVMs arise from a first-order stochastic difference equation, where its error term must be unforecastable with regard to past information, i.e., it must have a zero conditional expectation. This is exactly what the common cyclical feature framework implies. If this fails, the PV equation will not be valid, since it will contain an additional term that captures the (non-zero) conditional expected value of all future error terms. Cointegration imposes the transversality condition, allowing the limit $I(0)$ combination of $Y_t$ and $y_t$ to be discarded. The existence of an unforecastable linear combination of the $I(0)$ series in the difference equation guarantees that the dynamic behavior of the variables in the PVM will conform to theory.

Since we need both conditions in order to validate PVMs, we will ideally work with an integrated econometric framework that encompasses the joint existence of these two phenomena. This is the starting point of this article. We first show that PV relationships entail a weak-form common feature restriction, as per Athanasopoulos et al. (2011) and Hecq, Palm, and Urbain (2006), for the vector error-correction model (VECM) for $Y_t$ and $y_t$. Alternatively, they also imply a polynomial serial correlation common feature relationship (Cubadda & Hecq, 2001) for the VAR representations of $\Delta y_t$ and the cointegrating relationship $Y_t - \theta y_t$. These represent short-run restrictions on the dynamic system for these variables. Once we cast the PVM in these terms, it is straightforward to apply the toolkit of the *common-feature* literature to inference and testing.

Our second contribution relates to the forecasting of series that are subject to PVM restrictions. We show the relevance of the issues discussed above in an empirical exercise involving two sets of financial series. The first contains annual long- and short-maturity interest rates for the US economy. The second contains real prices and dividends for the S&P composite index and the real risk-free rate. Both data sets were extracted from the online library maintained and updated by Shiller (http://www.econ.yale.edu/~shiller/data.htm), with 142 annual observations spanning the period 1871–2012. We are careful to consider the different layers of restrictions discussed in the PVM literature: long-run restrictions (cointegration), short-run restrictions (weak-form common cycles), long- and short-run restrictions jointly, and the last with additional specific parameter restrictions implied by economic theory. Each layer corresponds to a specific restricted representation for the reduced form VAR/VECM. Forecast accuracy measures are compared across representations in order to evaluate the benefits of imposing each set of restrictions. The final results confirm the importance of imposing short-run restrictions. Indeed, for target variables in PVMs ($Y_t$), forecasting models that allow for and/or impose these restrictions produce winners in 70% of cases at horizons from one to five years ahead. Overall, for $Y_t$ and $y_t$, they produce winners 63.33% of the time at these horizons.

Our last contribution is to devise a testing strategy for PV restrictions in macroeconomics and finance, incorporating more than 20 years of research on this topic. We cover several important issues. First, we discuss how to choose the lag length of the VAR consistently. Second, we discuss how to test for cointegration, common cycles, and weak-form common cycles, using a multivariate approach based on the likelihood ratio test (canonical correlation analysis) and a single-equation heteroskedasticity robust approach (GMM). Part of our suggested strategy relies on Monte-Carlo simulation results. Finally, we also suggest integrated approaches estimating the lag length of the VAR and the long-run and short-run parameters jointly, as per Athanasopoulos et al. (2011). Alternatively, we also discuss the joint estimation of the long-run and short-run parameters, as per Centoni, Cubadda, and Hecq (2007). In order to avoid taking up too much space in a forecasting paper with testing and estimation issues, these are discussed in Appendix A.

The rest of the paper is arranged as follows. Section 2 reviews PV formulas (for both the levels and log-levels of the variables) and discusses the types of restrictions that PVMs imply for the VECM, as well as for a transformed VAR. In Section 3, we present an in-sample analysis of the data used in the forecasting experiment, to verify whether the restrictions implied by economic theory hold in practice. In Section 4, we compare the forecasting gains obtained in multivariate models by imposing different types of PV restrictions. Section 5 concludes. Appendix A contains additional material on how to select the lag-length of the VAR in our context, how to implement different tests for PVMs, including their small-sample performances, and other issues that are relevant when examining PVM restrictions. We also present an online appendix (see Appendix B) including only self-contained material on common-cyclical features for cointegrated data.

## 2. Present-value models

### 2.1. *Nesting the representation in levels with long- and short-run co-movement*

Consider the present value equation $Y_t = \theta(1-\delta)\sum_{i=0}^{\infty}\delta^i \mathbb{E}_t y_{t+i}$, where we drop the constant term for

[1] Examples of $Y_t$ and $y_t$ include prices and dividends for a given asset, long- and short-term interest rates, and consumption and disposable income, respectively. If they are integrated processes, they will be cointegrated. See also the examples from Campbell (1987) and Campbell and Deaton (1989), *inter alia*, which are reviewed by Engsted (2002); together with the interesting recent contribution by Johansen and Swensen (2011).