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Optimizing multi-item multi-period inventory control system with discounted cash flow and inflation: Two calibrated meta-heuristic algorithms

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ABSTRACT

A mixed binary integer mathematical programming model is developed in this paper for ordering items in multi-item multi-period inventory control systems, in which unit and incremental quantity discounts as well as interest and inflation factors are considered. Although the demand rates are assumed deterministic, they may vary in different periods. The situation considered for the problem at hand is similar to a seasonal inventory control model in which orders and sales happen in a given season. To make the model more realistic, three types of constraints including storage space, budget, and order quantity are simultaneously considered. The goal is to find optimal order quantities of the products so that the net present value of total system cost over a finite planning horizon is minimized. Since the model is NP-hard, a genetic algorithm (GA) is presented to solve the proposed mathematical problem. Further, since no benchmarks can be found in the literature to assess the performance of the proposed algorithm, a branch and bound and a simulated annealing (SA) algorithm are employed to solve the problem as well. In addition, to make the algorithms more effective, the Taguchi method is utilized to tune different parameters of GA and SA algorithms. At the end, some numerical examples are generated to analyze and to statistically and graphically compare the performances of the proposed solving algorithms.

1. Introduction and motivation

One of the models that have been applied extensively to solve various productions and inventory control problems is the economic order quantity (EOQ). Many researches refer the history of EOQ to Harris [1]. This classic model encompasses the planning for one product in a period with several assumptions. Although these assumptions make the model simple, the usability of the EOQ model in real-world situations is limited.

From financial standpoint, an inventory represents a capital investment and must compete with other assets within the firm's limited capital funds. However, the traditional inventory models available in the literature do not take into account the time value of money (TVM) due to interest rate and inflation. Since these factors have significant effects on the economic

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order quantity (EOQ) of any inventory system and the resource of an industry is highly related to the return of investment and its time value, both inflation and TVM is very important in making managerial decisions. Buzacott [5] and Misra [26] extended various approaches for different inventory models with finite replenishments and shortages by considering the time value of money using different inflation rates for the costs. Chandra and Bahner [7] and Sarkar and Pan [37] developed infinite/finite replenishment models with shortages with inflation and TVM. Datta and Pal [11] investigated a finite horizon inventory model with time-dependent demand rate, shortages, inflation, and TVM.

More recent research work on the EOQ model contains multi-item multi-period inventory problems that were formulated to consider different factors such as inflation and TVM in real world situations. Das et al. [10] developed a multi-item inventory model with constant demand and infinite replenishment under the restrictions on storage area, total average shortage cost, and total average inventory investment cost. [40] developed a constrained optimization model for a group of end items with known constant demand rate and convertibility to other useful units. Kirkpatrick et al. [20] presented a mathematical model for managing inventory of a product in multiple periods. Further, many researchers investigated multi-item problems for deteriorating items [4,30,13]. Kim and Kim [19] formulated a multi-period inventory/distribution planning problem into a mixed integer linear programming and solved it by a Lagrangian relaxation approach. Goyal and Giri [14] presented a review article on the recent trends in inventory modeling of deteriorating items listing all the important publications in this area up to 2001. Panda et al. [29] proposed a nonlinear goal programming technique to obtain economic order quantities of multi-item inventory systems. Moreover, many researches worked on the multi-item Newsboy problem [1,2,6,3,38,41].

Quantity discount policies are usually treated as an inventory coordination mechanism between buyers and suppliers [39]. To name a few research works in this area, Chang and Chang [8] used a linear programming relaxation based on piecewise linearization techniques to solve an inventory problem with variable lead-time, crashing cost, and price-quantity discount. Maiti and Maiti [23] developed a problem for multi-item inventory control system of breakable items with all unit (AUD) and incremental quantity discounts (IQD). Taleizadeh et al. [42] developed a mixed-integer nonlinear programming model for a multiproduct multi-constraint inventory control problem with stochastic replenishment intervals and discount and solved it by a genetic algorithm. Further, Taleizadeh and Niaki [43] proposed a fuzzy mixed-integer nonlinear programming model for a multiproduct multi-chance constraints inventory problem with probabilistic period length and total discount under fuzzy purchasing price and holding cost and solved it by a hybrid meta-heuristic intelligent algorithm. Sana and Chaudhuri [36] extended the EOQ model by both permitting delay in payments and price discount.

As a population-based meta-heuristic method, genetic algorithm (GA) has been widely used in different areas such as scheduling [12], inventory classification [15], discounted multi-item inventory models [23,44], facility location problem [33], and supply chain management [25]. Mondal and Maiti [27] developed a multi-item fuzzy EOQ model and Maiti and Maiti [22] investigated damageable items in imperfect production processes. Both aforementioned papers utilized GA to solve their mathematical models. Another widely used meta-heuristic algorithm that has been used to solve sophisticated problems in different fields of study is the simulated annealing (SA) algorithm. Interested readers are referred for example to [32,33,42,44].

Since a major gap between the theoretical solutions of multiproduct multi-period inventory control and real world problems is still existed, the concentration of this paper is based on developing a more realistic model by infracting the traditional assumptions step-by-step to overtop the usability problem [31]. The proposed model is developed for an independent multi-product inventory system with known but inconstant demand for each product, an overall storage constraint, limited budget, and constrained order quantities. Moreover, not only TVM and inflation are assumed, but also specific all units discount (AUD) for some items and incremental quantity discount (IQD) for others are considered. These realistic situations encompass many limitations and enhance the applicability as well as the complexity of the model. The objective is to determine the optimal order quantities of the products such that the net present value of total system cost over a finite horizon is minimized. Since the model is overly NP-hard, two meta-heuristic algorithms of GA and SA along with a branch and bound (B&B) algorithm are presented to solve the proposed mathematical model. Moreover, to enhance the performances of the proposed meta-heuristic algorithms, a mechanism based on the Taguchi optimization technique is developed to calibrate the parameters of the two meta-heuristic algorithms. At the end, to analyze the performances of the proposed algorithms, some numerical examples are generated to compare the performances of the algorithms graphically and statistically.

The rest of the paper is organized as follows. In the next section the problem along with its assumptions are described. The mathematical formulation of the problem is given in Section 3. The B&B and the two meta-heuristic algorithms are discussed in Section 4 to solve the problem. The details of the parameter calibration come in Section 5. Computational results and comparisons are illustrated in Section 6. Section 7 contains conclusion and some recommendations for future research.

2. Problem definition and assumptions

Consider a company that stores several items to satisfy its customers' demands with known rates that may vary in different periods within a finite planning horizon containing N periods. The initial inventory of all items is zero, not more than one order can be placed for a specific item in a period, and that the order quantity of an item is limited. Besides, the ordered quantities of items are delivered in batch sizes where no split-batch is allowed. Depending on the order quantity, there is an all unit discount schedule to purchase some items and an incremental discount policy to purchase the others. Moreover, all the costs including the ordering, the purchasing, and the holding increase by a discount rate that is fixed during the