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An empirical model of fractionally cointegrated daily high and low stock market prices[☆]

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ABSTRACT

This work provides empirical support for the fractional cointegration relationship between daily high and low stock prices, allowing for the non-stationary volatility of stock market returns. The recently formalized fractionally cointegrated vector autoregressive (VAR) model is employed to explain both the cointegration dynamics between daily high and low stock prices and the long memory of their linear combination, i.e., the range. Daily high and low stock prices are of particular interest because they provide valuable information about range-based volatility, which is considered a highly efficient and robust estimator of volatility. We provide a comparison of the Czech PX index with other world market indices: the German Deutscher Aktienindex (DAX), the U.K. Financial Times Stock Exchange (FTSE) 100, the U.S. Standard and Poor's (S&P) 500 and the Japanese Nihon Keizai Shimbun (NIKKEI) 225 during the 2003–2012 period, that is, before and during the financial crisis. We find that the ranges of all of the indices display long memory and are mostly in the non-stationary region, supporting the recent evidence that volatility might not be a stationary process. No common pattern is detected among all of the studied indices, and different behaviors are also observed in the pre-crisis and post-crisis periods. We conclude that the fractionally cointegrated VAR approach allowing for long memory is an interesting alternative for modelling range-based volatility.

1. Introduction

Daily high and low stock market prices provide valuable information about range-based volatility that is not included in the open and close prices commonly studied by researchers. More specifically, the difference between high and low prices, i.e., the range, provides an efficient estimator of volatility robust to noise (Parkinson, 1980). To date, stock prices in developed markets have generally been considered to be unpredictable and are assumed to follow a random walk. Hence, most studies consider stock prices to be integrated of order 1 (an $I(1)$ process). However, the choice between stationary $I(0)$ and non-stationary $I(1)$ processes can be too restrictive for the degree of integration of daily high and low prices. Because high and low prices can be modeled together as a possibly fractionally cointegrated relationship (Cheung, 2007; Fiess and MacDonald, 2002), it allows for greater flexibility. This idea is especially interesting because the error correction term from the cointegrating relationship between high and low prices is the range. Hence, a more general fractional or long-memory framework, where the series are assumed to be integrated of order d and cointegrated of order less than d, i.e., $CI(d-b)$, where $d, b \in R$ and $0 < b \le d$, could be more useful in capturing the empirical properties of data.

To determine the fractionally cointegrated relationship between highs and lows, we implement a fractionally cointegrated vector autoregressive model (FCVAR), as proposed by Johansen (2008) and Johansen and Nielsen (2010, 2012). The motivation for utilizing this framework is twofold. First, daily highs and lows are assumed to be cointegrated, i.e., in the short term they may diverge, but in the long term they have an embedded convergence path. Second, their specific linear combination is an efficient volatility estimate, i.e., the range, and is assumed to display a long memory.

Substantial evidence of the presence of long memory has been documented in the literature not only in the volatility of asset prices (Ding et al., 1993; Andersen and Bollerslev, 1997; Breidt et al., 1998; Kellard et al., 2010; Garvey and Gallagher, 2012) but also in the interest rate differentials, inflation rates, forward premiums, and exchange rates (Baillie, 1996). Although the vast literature concludes that volatility is a long-memory process, few studies suggest that volatility is a non-stationary process with the long memory parameter d being greater or equal to 0.5 (Kellard et al., 2010). Yalama and Celik (2013) provide an excellent review of the literature studying

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the long memory properties of volatility and document the feature empirically as well.

This work contributes to the literature through an empirical investigation of world market indices, especially of their daily high and low prices, in the fractional cointegration framework. Their linear combination, the daily range, is found to be a non-stationary process. Whereas Caporin et al. (2013) suggest a fractionally cointegrated framework for modelling daily high and low prices in their pioneering work, we present new empirical evidence of the long memory behavior of global stock markets. Moreover, we add a long memory analysis utilizing different measures and different periods such as pre-crisis and crisis, and thus we present new empirical evidence.

The analysis is performed on four global stock market indices, the U.S. Standard and Poor's (S&P) 500, the German Deutscher Aktienindex (DAX), the Japanese Nihon Keizai Shimbun (NIKKEI) 225, and the U.K. Financial Times Stock Exchange (FTSE) 100 index over the 10-year period 2003–2012. These results are compared to the Czech PX Index[1] over the same period. Moreover, we study the behavior of the high and low prices in two sub-periods with December 2007 as the dividing point. This analysis enables us to compare both cointegration and estimated volatility before and during the recent financial crisis. The main result is that we find significant evidence of long memory in the daily ranges falling in the non-stationary region (except for the PX Index and NIKKEI 225 in the first period). Our results also distinguish between the two sub-periods. The long memory estimates during the first period of 2003–2007 are generally lower in comparison to the second period, where primarily the years 2008 and 2009 seem to increase the long memory. Overall, the PX Index displays the lowest estimates of the order of price range integration, and its behavior is very similar to the NIKKEI 225 in this respect. The ranges of the S&P 500, FTSE 100 and DAX indices display, however, relatively higher orders of integration. Furthermore, we find that the unrestricted FCVAR performs better in detecting the stationarity of the range indicated by other applied tests than the FCVAR specification with restrictions on the cointegrating vector.

The remainder of the study is organized as follows. Section 5 describes the motivation for using daily high and low prices and descriptions of the data. In Section 3, we conduct the preliminary analysis of daily high and low prices and the range, focusing on their long memory properties. Section 4 then suggests an empirical model of fractionally cointegrated daily high and low prices and discusses the main results. Finally, Section 5 concludes the findings.

2. Motivation and data description

In this work, we focus on investigating daily high and low prices. By the high price, we understand the maximum price observed during the day, and by the low price, we understand the minimum price achieved during that day. These prices can be viewed as additional information about the change in direction of excess demand (Cheung, 2007). Caporin et al. (2013) nicely summarize other reasons why the daily high and low prices are of importance. First, daily high and low prices can have a role as a reference level. Stock market agents employ these reference levels to make assumptions and predictions about future developments and employ daily highs and lows as reference values. Second, daily highs and lows can function as a stop-loss indicator and may contain information about liquidity provisioning and the price discovery process. Third, high and low prices are more likely to correspond to ask and bid quotes, respectively, implying that they may be influenced by transaction costs and other market frictions (e.g., price discreteness, stale prices, and tick size). Moreover, daily high and low prices tend to react to unanticipated public announcements or other unexpected shocks.

Daily high and low stock prices are primarily valuable as a measure of dispersion, i.e., a measure of the deviation from the mean. In financial literature, dispersion measures the degree of uncertainty, and thus risk, associated with a particular asset. Parkinson (1980) was among the first to show that a variance estimator based on close-to-close returns is a far less efficient volatility estimator than the price range defined as a difference between daily high and low prices. Alizadeh et al. (2002) further demonstrate that a range-based estimator of volatility is highly statistically efficient and robust with respect to several types of microstructure frictions because it is much less contaminated by measurement error and explains not only the autocorrelation of volatility but also the volatility of volatility. Furthermore, Corwin and Schultz (2012) argue that, because daily high and low prices are mostly buy and sell trades, respectively, the price range thus represents a fundamental volatility because it reflects both the stock's variance and its bid–ask spread. Alizadeh et al. (2002) note that using the range as a volatility proxy has a "long and colorful" history in finance (e.g., Andersen and Bollerslev, 1998; Degiannakis and Livada, 2013; Garman and Klass, 1980; Parkinson, 1980). More recently, Caporin et al. (2013) find evidence of long memory in the ranges of all 30 of the components of the Dow Jones Industrial Average (DJIA) index during the 2003–2010 period.

This work focuses on an analysis of the daily high and low prices of four major global indices over the 2003–2012 period covering both the calm and the financial crisis periods. We consider four world indices: the U.S. S&P 500, the German DAX, the Japanese NIKKEI 225, and the U.K. FTSE 100, available from TICK data, which are examined and compared to the Czech stock market index. Moreover, we examine the indices during the entire 10-year period from January 2003 to December 2012 and as during two sub-periods. The first sub-period covers the pre-crisis years from January 2003 until December 2007. This break point has been chosen based on the statement of the National Bureau of Economic Research (NBER), which identified December 2007 as the peak of pre-crisis economic activity. The subsequent decline in economic activity was large enough to be qualified as a recession. The second sub-period spans from January 2008 until December 2012 and covers the recent crisis period. We synchronize the data with the same time stamps but discard holidays from further analysis.

Fig. 1 depicts the development of daily high and low prices of the PX index and their difference – the range. The peak of December 2007 is depicted as a vertical line in the figure. We can see that the PX index was experiencing steep growth in the first period, but at the end of 2007, it was severely hit by the crisis. After a stable period from mid-2009 through mid-2011, another drop followed; however, it was less dramatic than the decline observed at the end of 2007.

The range-based volatility measured as the difference between daily high and low prices is higher after the outbreak of the crisis and reaches its maximum at the end of 2008; then it gradually returns to its pre-crisis values. A similar pattern emerges for all of the other studied indices.[2] With the DAX, we document a slightly different behavior, mainly after the crisis, when it grows steadily following the drop in 2007, with the exception of a short period of decline at the end of 2011. This pattern is also the case with the S&P 500. However, the FTSE 100 displays only a very slight or no growth after the crisis, and the NIKKEI 225 actually declines.

When we compare the behavior of all of the studied markets, we find that they are influenced by similar factors; however, the reaction to the crisis is quite different for each of them. All of the indices experienced rapid growth during the 2003–2007 period followed by a steep downturn at the end of 2007. The DAX and PX indices are the two indices least affected by the crisis. The DAX is also the first index in our

[1] The PX Index was introduced in March 2006 as a merging of two indices, PX-D and PX 50. The PX Index obtained the historical prices from the PX 50 and continued henceforth.

[2] Figures for all of the remaining series can be found in the Appendix A (Fig. 3 for the DAX, Fig. 4 for the FTSE 100, Fig. 5 for the NIKKEI 225, and finally, Fig. 6 for the S&P 500).