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Stock price dynamics and option valuations under volatility feedback effect

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ABSTRACT

According to the volatility feedback effect, an unexpected increase in squared volatility leads to an immediate decline in the price–dividend ratio. In this paper, we consider the properties of stock price dynamics and option valuations under the volatility feedback effect by modeling the joint dynamics of stock price, dividends, and volatility in continuous time. Most importantly, our model predicts the negative effect of an increase in squared return volatility on the value of deep-in-the-money call options and, furthermore, attempts to explain the volatility puzzle. We theoretically demonstrate a mechanism by which the market price of diffusion return risk, or an equity risk-premium, affects option prices and empirically illustrate how to identify that mechanism using forward-looking information on option contracts. Our theoretical and empirical results support the relevance of the volatility feedback effect. Overall, the results indicate that the prevailing practice of ignoring the time-varying dividend yield in option pricing can lead to oversimplification of the stock market dynamics.

1. Introduction

The fundamental importance of time varying volatility has long been recognized in statistical finance and financial economics, and many scientific findings thereof have been well accepted and exploited in these disciplines. The topic is motivated by strong evidence that volatility does not remain constant over time. Recently, in October 2008, the volatility index of S&P 500, VIX, hit 80% whereas its average (1990–2009) was about 20%. Empirically, also the price–dividend ratio (or its reciprocal, the dividend yield) is time-varying and one of the 'stylized facts' of financial markets is that changes in the price–dividend ratio are negatively correlated with volatility. Many theories, of which the so-called volatility feedback effect (sometimes called the risk-premium effect) is one, explain the empirically observed negative correlation between volatility and stock price (see, e.g., Refs. [1–8]). According to the theory of volatility feedback effect, an unexpected increase in squared volatility leads to an immediate decline in the stock price, because cash flows are discounted at a higher rate. Therefore, an exogenous increase in squared volatility generates additional return volatility as stock prices respond and adjust to new information about the cost of capital. In addition, the relation between volatility and returns can (at least partly) be explained by the leverage effect, which extends from changes in the firm's value to changes in stock returns and volatility. The difference lies in causality—the volatility feedback effect theory contends that changes in volatility may produce return shocks, whereas the leverage hypothesis predicts that return shocks lead to changes in volatility. Also the leverage effect is widely examined in the literature (see, e.g., Ref. [9] and references therein).

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The time-varying price–dividend ratio (or the dividend yield) and its relation to stochastic volatility is well documented in the empirical literature, but the current option pricing literature does not sufficiently characterize the joint dynamics of dividends, volatility, and stock price; instead, typically in option pricing, dividends are either ignored or the dividend yield is assumed to be constant at best. In this paper, we aim to show that the prevailing practice of ignoring the modeling of the joint dynamics of dividends, volatility, and stock prices is inconsistent not only with respect to financial data but also with respect to financial theory itself. This oversimplification can lead to mispricing of options and a misestimation of the effects of the return risk and volatility risk on option values. Our main goals are as follows:

- Model the joint stochastic dynamics of return volatility, dividends, and stock price with volatility feedback in continuous time by determining the underlying stock as a claim for future random dividends with a stochastic discount rate.
- Express the relation between dividend growth volatility and return volatility and solve the volatility puzzle (i.e. return volatility is too high compared to dividend growth volatility).
- Show that the correlation between returns and volatility can be divided into two components: leverage effect and volatility feedback effect.
- Demonstrate a mechanism by which the market price of return risk, or equity risk-premium, affects option prices.
- Show that, contrary to the prevailing view, an increase in squared return volatility can *negatively* affect the price of deep-in-the-money call options.
- Illustrate how to obtain forward-looking estimates for the price of diffusion return risk using information on option contracts.

One of the main implications of Black–Scholes theory is the irrelevancy of the equity risk premium in option valuation (i.e. option values are not functions of the expected rate of return). We, however, aim to show that the price of return risk determines the sensitivity of the dividend yield to return volatility and thereby affects option valuations, and consequently, the market price of return risk is needed as an input to price options under our framework. This, on the other hand, allows us to produce forward-looking option-implied estimates for the market price of the diffusion return risk and the volatility risk premium as a part of the calibration procedure of our model. These option-based estimates can be obtained using option data alone without the need of historical stock price data, which is in contrast to the traditional literature that usually uses a series of equity market indices producing backward-looking estimates for the market price of diffusion return risk.

In the early literature, Carr and Wu [10] provided a welcome exception by proposing a model that aims to capture the volatility feedback effect and estimate the jump risk and the variance rate risk using option data. Bakshi and Wu [11] specified a model to estimate market prices of different sources of risks using information on both time-series returns and options prices. However, the approaches in these two papers differ markedly from ours. Most importantly, they assume a constant dividend yield, an assumption that contradicts the empirical evidence of varying price–dividend ratio and the theory of the volatility feedback effect. Moreover, Carr and Wu [10] try to capture the volatility feedback effect directly by assuming a negative statistic correlation between business risk and stock price without modeling the changes that the underlying asset price undergoes in volatility. In addition, in contrast to our paper, the price of the *diffusion* return risk does not appear in option pricing formulas under the risk-neutral measure in Refs. [10,11]. Our paper is also related to Ref. [12], which integrates the stochastic dynamics of interest rates, dividends, and stock prices and valuates options accordingly. Despite of some methodological similarities between the papers, [12] differs substantially from the present paper; whereas Kanniainen [12] focuses on the joint dynamics of spot rate and dividends but ignores the volatility feedback effect, in this paper we investigate the stock market dynamics and options prices under the volatility feedback but, for simplicity, assume constant interest rates.

The paper is organized as follows. In Section 2, we present our model setup, solve the price–dividend ratio with it, and study stock market dynamics under our assumptions. In Section 3, we show how to price options under our settings, and in Section 4 we provide an empirical illustration. The final section discusses the results and draws conclusions.

2. Stock market dynamics

2.1. Model setup

Let $\{P_t; t \geq 0\}$ denote the stock price process and $\{D_t; t \geq 0\}$ the instantaneous dividend stream and let us assume that both $\{P_t; t \geq 0\}$ and $\{D_t; t \geq 0\}$ evolve on $\mathbb{R}_+$. We define the cumulative stock returns as follows:

Definition 1 (*Cumulative Stock Return*)**.** The cumulative stock return from dividends and changes in prices satisfies

$$\mathrm{d}R_t = \frac{\mathrm{d}P_t + D_t \mathrm{d}t}{P_t}.$$

Thus $\{R_t; t \geq 0\}$ represents the instantaneous total return including price appreciations and dividends. To focus on the characterization of stock market dynamics and valuation of options with volatility feedback condition, and to maintain conciseness and readability, we employ pure diffusion-based models and leave extensions, including jumps and non-affine volatility models, for future research. In the following, we characterize the dynamics of cumulative stock returns and return volatility.