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An economic manufacturing quantity model with probabilistic deterioration in a production system

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ABSTRACT

This paper develops an *economic manufacturing quantity* (EMQ) model with deterioration and exponential demand in a production system over a finite time horizon under the effect of inflation and time value of money. The production rate is a dynamic variable (varying with time) in a production system. Due to a long run process, the machinery system is converted from *in-control* state to *out-of-control* state which results the production of improper items. The improper items are reworked at a fixed cost to make it as proper. With the increasing value of time, the production of improper item also increases. To reduce the production of the improper items, the systems have to be more reliable and with less amount of failure. In this direction, the model considers that the development cost, production cost, and material cost are dependent on the reliability parameter. The deterioration of the product is considered probabilistic to make the research a more realistic one. By considering the reliability parameter as a decision variable, we try to obtain the associated profit of the system which we have to maximize. To derive the maximization procedure, we use Euler–Lagrange formula from control theory. We outline some numerical examples along with graphical representations and sensitivity analysis to illustrate the model.

1. Introduction

Generally, industrial engineers have given more importance to the study of the inventory model in different directions. Their inventory models basically depend on different issues like shortages, variable demand, inflation, deterioration, reliability, constraint, etc. The objectives of their models are to find either the maximum profit or minimum associated cost of the system. Some of them try to find the improved algorithms for the minimum cost or maximum profit. Comparing with the real world situation, it is very difficult to create an accurate model of the real life situations. For that purpose, in the Notation and assumptions section most of the parameters are considered as given or constant. But, it is not true always. Hence, we generally want to incorporate new ideas to an existing model and try to find the optimal cost. Those new ideas come from the real world system which contains different types of production systems, manufacturing companies, etc.

In the inventory literature, there are numerous numbers of models by considering the production process. Due to the long-run process, the machinery systems shift to *out-of-control* state, as a result, the systems produce imperfect items which cannot be sold directly. The imperfect items are reworked at a fixed cost to make them into new ones. Depending on this policy, Porteus (1986) developed a failure-prone production process. He discussed quality improvement and setup cost reduction in his model. Ben-Daya (1999) found out an integrated-production maintenance and quality improvement model for an imperfect production process. Hariga and Benkherouf (1994) discussed some optimal and heuristic replenishment models for deteriorating items with an exponentially time-varying demand. Wee (1995) studied an EOQ model allowing shortages where demand declined exponentially over time. Chung and Hou (2003) extended the EMQ model with imperfect production process and shortages. Sana et al. (2007) derived an excellent model on the imperfect production process for the volume flexible inventory model. Cárdenas-Barrón (2007) derived a technical note on optimal manufacturing batch size with rework process at a single-stage production system. Sana and Chaudhuri (2008) developed an inventory model for stock with an advertising sensitive demand. Chung and Wee (2008) extended the imperfect production model with inspection policy and warranty period. Cárdenas-Barrón (2008) found out a simple derivation on optimal manufacturing batch size with rework in a single-stage production system. Cárdenas-Barrón (2009) discussed an economic production quantity model with rework process at a single-stage manufacturing system with planned backorders. In this direction, Sana (2010a, 2010b, 2010c, 2010d) discussed different types of inventory models by considering a demand

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influenced by enterpriser's incentives, time varying deterioration and partial backlogging and imperfect production. Sarkar et al. (2010a) developed the joint determination of optimal production lot size, safety stock and reliability parameter. That model was extended by Sarkar et al. (2010b) under the realistic assumption that the production facility was subject to random breakdown of the machinery system. In that model, they showed that the reliability of a machinery system could be increased by more investments in production technology. Sarkar et al. (2010c, 2011a) discussed the EMQ model with stock and time dependent demands. Sana and Chaudhuri (2010) developed an EMQ model in an imperfect production process. Yang et al. (2011) discussed about mitigating hi-tech product risks due to rapid technological innovation. Sarkar et al. (2011b) derived an EPQ (economic production quantity) model with a stochastic demand for the imperfect production system. This model was again extended by Sarkar and Moon (2011) with the effect of inflation. Based on salesmen's initiatives and a stock sensitive demand Sana (2011) discussed an EOQ (economic order quantity) model for homogeneous products. Sarkar (2012a) developed an EOQ model with delay in payments and stock dependent demand in the presence of an imperfect production.

For the most realistic formulation of the mathematical model, we consider the probabilistic deterioration. Deterioration indicates the decay of the products like any foods, volatile liquids, medicines, etc. Hence, it preserves a vital role in most of industry/education/banking sector even also in the medical sectors, etc. The researchers are still trying to formulate the realistic model to solve the present problem of the above mentioned sectors. In this direction, enormous numbers of research are carried on. Among those, most of the researchers like Ghare and Schrader (1963), Goyal (1987), Chang and Dye (1999), Manna and Chaudhuri (2006), Sana (2010b) and others, developed different types of realistic models by considering deterioration. Chung and Wee (2007) found out a scheduling and replenishment plan for an integrated deteriorating inventory model with a stock dependent selling rate. Yang et al. (2010) established an EOQ model based on the sequential and global optimization for an inventory model with deterioration. Widyadana et al. (2011) developed an EOQ model with deteriorating items and planned backorder level. Widyadana and Wee (2011) discussed an optimal deteriorating items production inventory model with random machine breakdown and stochastic repair time. Sarkar (2012b) found out an EOQ model with delay in payments and a variable deterioration rate. Sarkar (2012c) derived a probabilistic deterioration model to find the integer number of deliveries and lot size with the help of an algebraical procedure. Recently, some notable research in the direction of deterioration are done by Sett et al. (2012), Sarkar et al. (2013), Sarkar and Sarkar (2013).

In this model, we develop a production model which produces both perfect and imperfect items. The imperfect items are reworked at a cost to make it as the original one. To make the system more reliable, the development cost, material cost, and unit production cost are considered as dependent on the reliability parameter. Keeping in mind the exponential nature of the market demand, the demand of this model is considered as exponential. Different types of probabilistic deteriorations of the products are considered in this model. The associated profit is maximized with the help of the Euler–Lagrange theory. After a long survey of the literature, it is found that an *EMQ* model with a variable reliability parameter and the probabilistic deterioration of products along with an imperfect production is quite rare. To the authors' best knowledge, no researchers have considered this type of model with a variable reliability parameter, a probabilistic deterioration of a product and with the presence of an imperfect production of items under the effect of inflation and time-value of money. The rest of the paper is organized as follows: notation and assumptions are considered in Section 2. The model formulation and analysis of the model is discussed in Section 3. Some numerical experiments are pointed out to illustrate the model in Section 4. Finally, concluding remarks are given in Section 5.

2. Notation and assumptions

The following notation and assumptions are used to develop the model:

Notation:

θ	probabilistic deterioration rate of the product;
$P(t)$	production rate of the system at time t which depends on the reliability parameter of the machinery system;
$D(t)$	time dependent demand rate at $t\geq 0$;
$Q(t)$	on-hand inventory at time $t\geq 0$;
$\dot{Q}$	derivative of the on-hand inventory $Q(t)$ with respect to time t;
ψ	system reliability parameter (decision variable);
$C_D(\psi)$	the development cost for the production system which varies with ψ;
$C_M(=C_0-C_1\psi)$	material cost per unit item which depends on ψ, $C_0>0$, $C_1\geq 0$;
α	variation constant of tool/die costs;
$C_P(\psi,t)$	production cost per unit item;
C_h	holding cost per unit per unit time;
ω	selling price per unit item sold;
R	rework cost per unit improper item;
$\delta=r-i$	where $r=$interest per unit currency and $i=$inflation per unit currency;
T	the duration of production-inventory cycle;
C_2	deterioration cost per unit item per unit time;
$\psi_{\max}$	maximum value of ψ;
$\psi_{\min}$	minimum value of ψ;

Assumptions:

1 The variable production rate of the system is always greater than the demand of the product. Therefore, there is no chance for shortage.
2 Deterioration rate is probabilistic.
3 The initial and final inventories are as $Q(0)=Q(T)=0$.
4 This model considers a production system which produces two types of product, proper and improper. Without any further investment, the proper items are ready for sale but the improper items are not directly ready for going to the market. There are some fixed rework costs which transfer the improper items to proper items.
5 Demand rate is considered as time-dependent i.e., $D(t)=ae^{bt}$, a, $b>0$;
6 The reliability parameter ψ of the production system is defined by $\frac{\text{Number of failures}}{\text{Total number of working hours}}$. ψ may be changed by new technology, resource of production system, etc.
7 The material cost $C_M=C_0-C_1\psi$ is dependent on reliability parameter ψ of the production system, $C_0>0$, $C_1\geq 0$. If $C_1=0$ the minimum material cost is C_0.
8 The unit production cost and the development cost are the function of system reliability parameter and production rate $P(t)$.
9 The effects of inflation and time value of money are considered.
10 Time horizon is finite.

3. Mathematical formulation and analysis of the model

We consider a production system which produces proper and improper items. To gain more profit, all companies want to produce more items and sell more of their products. But, if the machinery systems have not the sufficient capacity to produce more at a fixed time then either they have to arrange a redundant machinery system or they have to try to use their machinery systems for a long-run. Generally, the companies prefer to use the long-run process than buying