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Computing present values: Capital budgeting done correctly ☆

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ABSTRACT

This paper shows that the standard textbook formula for computing the present value of a future random cash flow – the discounted expected value – is formally incorrect and can generate significant errors when used to compute present values. The correct present value method is provided as well as a simple adjustment to the textbook formula which can be used to obtain an approximation to the correct value.

1. The introduction

Perhaps the most fundamental concept in corporate finance and capital budgeting is the notion of a present value. Computing a present value is one of the first tasks that a student of finance is expected to master. Therefore, it may be surprising to learn that the standard textbook formula for computing the present value of a future random cash flow – taking the discounted expected value of the cash flow (see Ross et al. (2008), chap. 12; Brealey and Myers (2003), chap. 9; Brealey et al. (2011), chap. 10; Berk and DeMarzo (2014), chaps. 18 and 19) – is formally incorrect and often generates significantly biased values.

☆ Helpful comments from Arkadev Chatterjea and Andrew Karolyi are greatly appreciated.

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The purpose of this paper is to prove this claim by: (1) showing the correct method, (2) documenting the size of the error when using the standard textbook formula, and (3) providing an adjustment to the textbook formula which, as an approximation, removes this error. The error in the textbook formula is due to the fact that it ignores the correlations between the cash flows and the discount rates when computing present values.

The goal, of course, is to improve decision making in standard capital budgeting procedures. Although many of these results are provable from existing equilibrium asset pricing models, I could find no source presenting these results. A minor contribution here is proving these results using only the absence of arbitrage.

An outline of the paper is as follows. Section 2 formalizes the argument for a discrete time model. Section 3 repeats the analysis for a continuous time model. This section contains the documentation of the size of the errors in using the textbook formula as well as the adjustment to obtain an approximation to the correct value. Section 4 concludes. All proofs are contained in the Appendix A.

2. The discrete time model

The discrete time model is presented first because the mathematics is simpler than in the continuous time case, and consequently the intuition behind the correct present valuation formula is clear.

We consider a finite horizon model where time increments in units, $t = 0, 1, \ldots, T$. The economy's randomness is characterized by a filtered probability space $(\Omega, \mathcal{F}, (\mathcal{F}_t)_{t\in[0,1,\ldots,T]}, \mathbf{P})$ satisfying the usual conditions, see Protter (2005), with $\mathbf{P}$ the statistical probability measure.

Let C_T be a random cash flow at time T assumed to be $\mathcal{F}_T$- measurable that we want to compute its present value. We assume that the cash flow is non-zero and strictly positive with positive probability, i.e. $\mathbf{P}(C_T \geqslant 0) = 1$ and $\mathbf{P}(C_T > 0) > 0$.

Let V_t be the present value of the cash flow at time t. Note that by definition $V_T = C_T$.

Assumption (No Arbitrage) $E(|V_t|) < \infty$ and $V_t > 0$ for all t.

This is a *no arbitrage* assumption because if V_t traded and for some t either $V_t = \infty$ or $V_t = 0$, then an arbitrage opportunity would exist. Indeed, in the first case (∞) buying V_t and holding the position until time T generates the arbitrage opportunity, and in the second case (0) shorting V_t and holding the position until time T generates the arbitrage opportunity.

Next, define μ_{T-1} to be the time $T-1$ conditional expected return on the cash flow's present value, i.e.

$$E_{T-1}\left(\frac{V_T}{V_{T-1}}\right) = 1 + \mu_{T-1}. \tag{2.1}$$

Note that when viewed at a time $t < T-1$, μ_{T-1} is a random variable. We call μ_{T-1} the *discount rate*. The no arbitrage assumption guarantees[1] both that μ_{T-1} exists and that $\mu_{T-1} > -1$.

Expression (2.1) implies that $V_{T-1} = E_{T-1}\left(\frac{V_T}{1+\mu_{T-1}}\right)$. Next, using the same logic at time $T-2$, we obtain $V_{T-2} = E_{T-2}\left(\frac{V_{T-1}}{1+\mu_{T-2}}\right)$.

Substitution of V_{T-1} into this last expression and using the law of iterated expectations gives $V_{T-2} = E_{T-2}\left(\frac{E_{T-1}\left(\frac{V_T}{1+\mu_{T-1}}\right)}{1+\mu_{T-2}}\right) = E_{T-2}\left(\frac{V_T}{[1+\mu_{T-2}][1+\mu_{T-1}]}\right)$.

Continuing, we get the present value formula.[2]

$$V_t = E_t\left(\frac{V_T}{[1+\mu_t][1+\mu_{t+1}]\cdots[1+\mu_{T-1}]}\right). \tag{2.2}$$

[1] The proof of this statement is simple. $V_T > 0$ and $V_{T-1} > 0$ imply that $\frac{V_T}{V_{T-1}} > 0$ so that $\frac{V_T}{V_{T-1}} - 1 > -1$. Taking the conditional expectation proves the claim.

[2] This is really a version of the Doob decomposition theorem, see Follmer and Schied (2004), p. 277.