International Communications in Heat and Mass Transfer 67 (2015) 39–45

Contents lists available at ScienceDirect

International Communications in Heat and Mass Transfer

journal homepage: www.elsevier.com/locate/ichmt

Solution of the two-dimensional inverse problem of the binary alloy solidification by applying the Ant Colony Optimization algorithm☆

Edyta Hetmaniok

Institute of Mathematics, Silesian University of Technology, Kaszubska 23, Gliwice 44-100, Poland

ARTICLE INFO

Available online 29 June 2015

Keywords:
Solidification
Binary alloy
Lever arm model
Swarm intelligence
ACO algorithm

ABSTRACT

In the paper a procedure for solving the two-dimensional inverse problem of the binary alloy solidification is presented. In considered problem the heat transfer coefficient is identified on the basis of measured values of temperature in selected points of the region. Direct problem, associated with the inverse one, is solved by using the generalized alternating phase truncation method. Functional expressing of the error of approximate solution is minimized with the aid of Ant Colony Optimization algorithm. The model also includes the macrosegregation, influencing the solidification, described by means of the lever arm model.

1. Introduction

During the last years a large popularity have been found by optimization techniques inspired by swarm intelligence and proceeding in the way simulating the behavior of swarms of social insects, flocks of birds, schools of fish, fields of weeds and so on. Expression “swarm intelligence” was formulated by Gerardo Beni and Jing Wang in 1989, in the context of cellular robotic systems [1] and it denotes the collective behavior of decentralized, self-organized, natural or artificial individuals. The advantage of such kind of procedures over the traditional optimization techniques lies in their flexibility and simplicity. These properties make the swarm intelligence algorithms the successful tool for solving optimization tasks appearing in many technical problems.

In this paper we deal with the problem of solidification of the alloys and ability of controlling this process on the way of determining the boundary conditions ensuring its best run. More precisely, the discussed problem consists in determination of the heat transfer coefficient appearing in the boundary condition defined on the boundary of two-dimensional region occupied by solidifying alloy. Another element, required to be reconstructed, is the distribution of temperature in the region of solidifying alloy. The sought elements will be calculated on the basis of temperature measurements read in selected points of investigated region.

Described problem belongs to the set of inverse problems that is the problems with incomplete mathematical description which should be reconstructed and completed [2]. The inverse problems are certainly much more difficult for solving than the direct problems in which the initial and boundary conditions are known. However, some methods for solving the inverse problem concerning the heat transfer and solidification have been proposed [3–9].

Solidification of the alloy is modeled by the, so called, solidification in the temperature interval, from the liquidus temperature to the solidus temperature. The mentioned model is founded on the heat conduction equation with the included source element in which the latent heat of fusion and the volume contribution of solid phase are included. Assuming the form of function describing this contribution we can transform the equation into the heat conduction equation with the so called substitute thermal capacity. Such reformulated differential equation defines the heat conduction in the entire homogeneous domain, that is in the solid phase, in the mushy zone (two-phase zone) and in the liquid phase [10–12].

Solidification of alloy is also influenced by the process of segregation of the alloy components, manifested for example in such a way that the change of concentration is associated with the change of liquidus and solidus temperatures. For describing this process we may use the model based on the lever arm rule in which the immediate equalization of chemical composition of the alloy in the liquid phase and solid phase is assumed [13–16]. Another model, also very often used for describing the concentration is the Scheil model [14,15,17,18].

In this paper we consider a model in which the distribution of temperature is described by means of the heat conduction equation [10–12] with the substitute thermal capacity and with the liquidus and solidus temperatures varying in dependence on the concentration of the alloy component. Whereas for describing the concentration we apply the model based on the lever arm rule. The direct solidification problem described by means of the used mathematical model, is solved with the aid of finite difference method supported by the alternating phase truncation method [19–22].

Important part of the proposed approach lies in the minimization of the functional representing the error of approximate solution. The tool

☆ Communicated by W.J. Minkowycz.

E-mail address: edyta.hetmaniok@polsl.pl.

http://dx.doi.org/10.1016/j.icheatmasstransfer.2015.05.029

Nomenclature

b	width of the spatial region
c_l	specific heat of the liquid phase
c_{mz}	specific heat of the mushy zone
c_s	specific heat of the solid phase
C	substantial thermal capacity
D	dimension of minimized problem
d	length of the spatial region
D_1	diffusion coefficients in the liquid phase
D_2	diffusion coefficients in the solid phase
f_j	contribution of solid phase in volume V_j
I	number of iterations in ACO algorithm
J	minimized functional
k	segregation coefficient
L	latent heat of solidification
m	number of ants in population
N_1	number of sensors
N_2	number of measurements
p^*	number of time nodes
s_{α_i}	standard deviation in α_i reconstruction
t	time
t^*	length of the time interval
T	temperature
T_0	initial temperature
T_{ij}	computed temperature
T_L	liquidus temperature
T_S	solidus temperature
T_∞	ambient temperature
U_{ij}	measured temperature
V_j	control volume
x	spatial variable
$\mathbf{x}^{best}$	best located ant
y	spatial variable
Z_0	initial concentration of the alloy
Z_L	concentration of the alloy component in liquid phase
Z_S	concentration of the alloy in solid phase

Greek symbols

α	heat transfer coefficient
β	narrowing parameter in ACO algorithm
Γ_i	boundary of the region
δ	relative percentage error
Δ	absolute error
Δr_j	area of the control volume
λ	thermal conductivity
λ_l	thermal conductivity of the liquid phase
λ_{mz}	thermal conductivity of the mushy zone
λ_s	thermal conductivity of the solid phase
ρ	mass density
ρ_l	mass density of the liquid phase
ρ_{mz}	mass density of the mushy zone
ρ_s	mass density of the solid phase
Ω	domain of the problem
$\overline{\Omega}$	spatial region

which will be used for executing this task is the Ant Colony Optimization algorithm (ACO algorithm), inspired by the behavior of real ants [23–26]. Application of optimization algorithms inspired by the nature in solving various kinds of inverse problems can also be found in papers [27–30]. In some earlier research, the author of the present papers, with her co-authors, has also used the swarm intelligence algorithm for solving the problems connected with heat conduction [31–37]. Some of these papers concern the one- and two-dimensional inverse problems of pure metal solidification modeled with the aid of Stefan problem (for example [31,37]), some of them discuss the solidification of alloys but in one-dimensional space (for example [35,36]). The current paper remains in the subject-matter of the author's interests and is devoted to the two-dimensional inverse problem of binary alloy solidification.

2. Formulation of two-dimensional problem

Let us investigate region $\overline{\Omega} = [0, b] \times [0, d] \subset \mathbb{R}^2$ occupied by the solidifying material. Boundary of region $\Omega = \overline{\Omega} \times [0, t^*]$ is divided into parts of the following form

$$\begin{aligned}
\Gamma_0 &= \{(x, y, 0);\ \ x \in [0, b],\ \ y \in [0, d]\},\\
\Gamma_1 &= \{(0, y, t);\ \ y \in [0, d],\ \ t \in [0, t^*]\},\\
\Gamma_2 &= \{(x, 0, t);\ \ x \in [0, b],\ \ t \in [0, t^*]\},\\
\Gamma_3 &= \{(b, y, t);\ \ y \in [0, d],\ \ t \in [0, t^*]\},\\
\Gamma_4 &= \{(x, d, t);\ \ x \in [0, b],\ \ t \in [0, t^*]\},
\end{aligned}$$

where the suitable initial and boundary conditions are defined. Two-dimensional domain of the problem in selected moment of time is presented in Fig. 1.

Information which is available for the researchers are the measurements of temperature in selected points of the domain, that is the values

$$T(x_i, y_i, t_j) = U_{ij},\ i = 1, 2, \ldots, N_1,\ j = 1, 2, \ldots, N_2, \tag{1}$$

where N_1 denotes the number of sensors and N_2 means the number of measurements taken from each sensor. On the basis of these data we need to identify are the value of heat transfer coefficient α on boundaries Γ_3 and Γ_4 as well as the distribution of temperature in region Ω. Function T, describing the distribution of temperature, must satisfy the heat conduction equation inside region Ω:

$$C\varrho \frac{\partial}{\partial t} T(x, y, t) = \lambda \nabla^2 T(x, y, t), \tag{2}$$

where C denotes the substitute thermal capacity and ρ and λ are, respectively, the mass density and thermal conductivity, whereas t describes the time variable and x, y refer to the spatial locations.

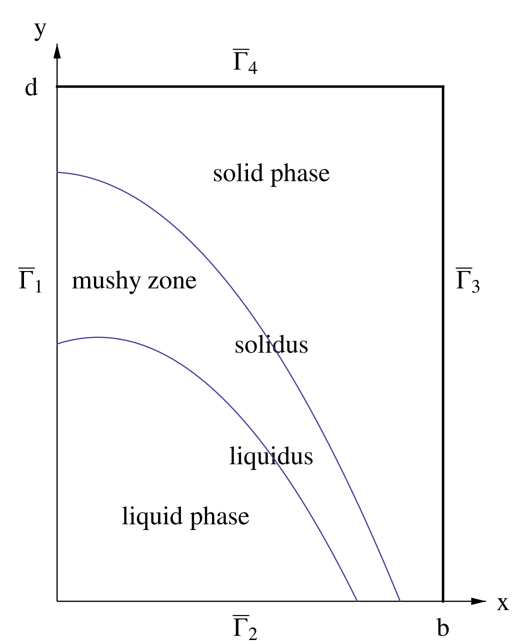

Fig. 1. Domain of the problem for the selected moment of time $\bar{t}$ ($\overline{\Gamma}_i = \Gamma_i \cap \{\bar{t}\}$).