



# Information cascade, Kirman's ant colony model, and kinetic Ising model



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## HIGHLIGHTS

- We show relations among our voting model, Kirman's ant colony model, and kinetic Ising model.
- Information cascade results from the quenching of the kinetic Ising model.
- When the candidates are above three, the voting model corresponds to Potts model.

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## ABSTRACT

In this paper, we discuss a voting model in which voters can obtain information from a finite number of previous voters. There exist three groups of voters:

- digital herders and independent voters,
- analog herders and independent voters, and
- tanh-type herders.

In our previous paper Hisakado and Mori (2011), we used the mean field approximation for case (i). In that study, if the reference number  $r$  is above three, phase transition occurs and the solution converges to one of the equilibria. However, the conclusion is different from mean field approximation. In this paper, we show that the solution oscillates between the two states. A good (bad) equilibrium is where a majority of  $r$  select the correct (wrong) candidate. In this paper, we show that there is no phase transition when  $r$  is finite. If the annealing schedule is adequately slow from finite  $r$  to infinite  $r$ , the voting rate converges only to the good equilibrium.

In case (ii), the state of reference votes is equivalent to that of Kirman's ant colony model, and it follows beta binomial distribution.

In case (iii), we show that the model is equivalent to the finite-size kinetic Ising model. If the voters are rational, a simple herding experiment of information cascade is conducted. Information cascade results from the quenching of the kinetic Ising model. As case (i) is the limit of case (iii) when tanh function becomes a step function, the phase transition can be observed in infinite size limit. We can confirm that there is no phase transition when the reference number  $r$  is finite.

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## 1. Introduction

While collective herding behaviour is popularly studied among animals, it can also be observed in human beings. In this regard, there are interesting problems across the fields of sociology [1], social psychology [2], ethnology [3,4], and economics.

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Statistical physics has been an effective tool to analyse these macro phenomena among human beings and has led to the development of an associated field, sociophysics [5,6]. For example, in statistical physics, anomalous fluctuations in financial markets [7,8] and opinion dynamics [9–16] have been discussed.

Most individuals observe the actions of other individuals to estimate public perception and then make a choice similar to that of the others; this is called social learning. Because it is usually sensible to do what other people are doing, collective herding behaviour is assumed to be the result of a rational choice that is based on public perception. While this approach could be viable in some ordinary cases, as a macro phenomenon, it can sometimes lead to arbitrary or even erroneous decisions. This phenomenon is known as an information cascade [17]. In this paper, we show that an information cascade is described by the Ising model.

In our previous paper, we introduced a sequential voting model [18]. At each time step  $t$ , one voter opts for either of two candidates. As public perception, the  $t$ th voter can see all previous votes, that is,  $(t - 1)$  votes. To identify the relationship between information cascade and phase transition, we introduce two types of voters—herders and independents. We also introduce two candidates.

The herders' behaviour is known as the influence response function, and threshold rules have been derived for a variety of relevant theoretical scenarios representing this function. Some empirical and experimental evidence supports the assumption that individuals follow threshold rules when making decisions in the presence of social influence [19]. This rule posits that individuals will switch from one decision to another only when sufficiently many others have adopted the other decision. Such individuals are called digital herders [20]. From our experiments, we observed that human beings exhibit a behaviour between that of digital and analog herders, that is, the tanh-type herder [21]. We obtained the probability that a herder makes a choice under the influence of his/her prior voters' votes. This probability can be fitted by a tanh function [22].

Here, we discuss a voting model with two candidates. We set two types of voters: independents and herders. As their name suggests, the independents collect information independently, that is, their voting depends on their fundamental values and rationality. In contrast, the voting of herders is based on public perception, which is visible to them in the form of previous votes. In this study, we consider the case wherein a voter can see the latest  $r$  previous votes.

When  $r \rightarrow \infty$  is the upper limit of  $t$ , we can observe several phenomena [20]. In the case where there are independent voters and digital herders, the independents cause the distribution of votes to converge to one-peak distribution, a Dirac measure when the ratio of herders is small. However, if the ratio of herders increases above the transition point, we can observe the information cascade transition. As the fraction of herders increases, the model features a phase transition beyond which a state where most voters make the correct choice coexists with one where most of them are wrong. Further, the distribution of votes changes from one peak to two peaks.

In the previous paper, we discussed the finite  $r$  case [20]. We analysed the model by using mean field approximations and concluded that information cascade transition occurs when  $r \geq 3$ . In this paper, we discuss the model from other perspectives and show that there is no phase transition when  $r$  is finite and the solution oscillates between two equilibria. Furthermore, we show relations among our voting model, Kirman's ant colony model, and kinetic Ising model.

The remainder of this paper is organised as follows. In Section 2, we introduce our voting model and mathematically define the two types of voters—independents and herders. In Section 3, we discuss the case where there are digital herders and independents. In Section 4, we verify the transitions between voting choices through numerical simulations. In Section 5, we discuss the case where there are analog herders and independents and show the relation between our voting model and Kirman's ant colony model. In Section 6, we discuss the relation between our voting model and the kinetic Ising model. In the final section, we present the conclusions.

## 2. Model

We model the voting of two candidates,  $C_0$  and  $C_1$ . The voting is sequential, and at time  $t$ ,  $C_0$  and  $C_1$  have  $c_0(t)$  and  $c_1(t)$  votes, respectively. In each time step, one voter votes for one candidate. Hence, at time  $t$ , the  $t$ th voter votes, after which the total number of votes is  $t$ . Voters are allowed to see just  $r$  previous votes for each candidate; thus, they are aware of public perception.  $r$  is a constant number.

We assume an infinite number of two types of voters—independents and herders. The independents vote for  $C_0$  and  $C_1$  with probabilities  $1 - q$  and  $q$ , respectively. Their votes are independent of others' votes, that is, their votes are based on their own fundamental values.

Here, we set  $C_0$  as the wrong candidate and  $C_1$  as the correct one in order to validate the performance of the herders. We can set  $q \geq 0.5$ , because we believe that independents vote for  $C_1$  rather than for  $C_0$ . In other words, we assume that the intelligence of the independents is virtually accurate.

In contrast, the herders' votes are based on the number of previous  $r$  votes. At time  $t$ , the information of  $r$  previous votes are the number of votes for  $C_0$  and  $C_1$ :  $c_0^r(t)$  and  $c_1^r(t)$ , respectively. Hence,  $c_0^r(t) + c_1^r(t) = r$  holds. If  $r > t$ , voters can see  $t$  previous votes for each candidate. For the limit  $r \rightarrow \infty$ , voters can see all previous votes. We define the number of all previous votes for  $C_0$  and  $C_1$  as  $c_0^\infty(t) \equiv c_0(t)$  and  $c_1^\infty(t) \equiv c_1(t)$ .

Now we define the majority's correct decision. If the ratio to the candidate  $C_1$  who is correct is  $c_1/t > 1/2 (< 1/2)$ , we define the majority as correct (wrong). This ratio is important to evaluate the performance of the herders. In this paper, we consider three kinds of herders, namely digital, analog, and tanh-type herders. We define  $z_r$  as  $z_r = c_1^r/r$ . The probability that a herder who refers  $z_r$  votes to the candidate  $C_1$  is defined as  $f(z_r)$ . Digital herders always choose the candidate with

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