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An interval type-2 fuzzy logic system-based method for prediction interval construction

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ABSTRACT

This paper introduces a new non-parametric method for uncertainty quantification through construction of prediction intervals (PIs). The method takes the left and right end points of the type-reduced set of an interval type-2 fuzzy logic system (IT2FLS) model as the lower and upper bounds of a PI. No assumption is made in regard to the data distribution, behaviour, and patterns when developing intervals. A training method is proposed to link the confidence level (CL) concept of PIs to the intervals generated by IT2FLS models. The new PI-based training algorithm not only ensures that PIs constructed using IT2FLS models satisfy the CL requirements, but also reduces widths of PIs and generates practically informative PIs. Proper adjustment of parameters of IT2FLSs is performed through the minimization of a PI-based objective function. A metaheuristic method is applied for minimization of the non-linear non-differentiable cost function. Performance of the proposed method is examined for seven synthetic and real world benchmark case studies with homogenous and heterogeneous noise. The demonstrated results indicate that the proposed method is capable of generating high quality PIs. Comparative studies also show that the performance of the proposed method is equal to or better than traditional neural network-based methods for construction of PIs in more than 90% of cases. The superiority is more evident for the case of data with a heterogeneous noise.

1. Introduction

Zadeh [1] introduced the type-2 fuzzy set (T2 FS) as an extension of the concept of an ordinary type-1 fuzzy set (T1 FS). Fundamental theories of the type-2 fuzzy logic system (T2 FLS), and in particular the interval T2 FLS (IT2FLS), were developed in late 90s and early 20s [2–5]. IT2FLSs are characterized by secondary membership functions (MFs) that only take the value of one over their domain. This significantly reduces the computational requirements needed for the type reduction stage. The extension of the type-1 FLS (T1 FLS) to its T2 counterpart allows the decision maker to account for more uncertainties present in data or operation of systems. The structure of an IT2FLS is similar to the structure of a T1 FLS. The only difference is the output processing block. In the T1 FLSs, this only includes a defuzzifier that produces a crisp output. The output processing block in IT2FLSs includes a type reduction unit that transforms a T2 FS into a T1 FS. This type-reduced set is then defuzzified into a crisp value [6].

In light of the work done by Mendel and his team and further studies conducted in recent years [7–9], it is already known that IT2FLSs are a promising tool for processing uncertain data. Often, both qualitative and quantitative (numerical) uncertainties in data may translate into rule uncertainties, destroying performance of a model. IT2FLSs with additional degrees of freedom make it possible to manage and minimize the effects of uncertainties, even better than traditional T1 FLSs [5]. The recent literature is rich in application of IT2FLSs in fields such as control [10], path planning [11], decision-making [12], and forecasting [13,14].

Research works on FLSs and their application for point forecast and prediction are abundant. However, there are two problems with those forecasts: (i) models become unreliable in the presence of uncertainty and (ii) no indication of accuracy of point forecasts is provided. One approach to quantify uncertainties associated with forecasts is to construct prediction intervals (PIs). Compared to point forecasts, PIs not only possess an indication of their accuracy, called the confidence level, but they also provide more reliable information about the future realization of the underlying

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target. A $(1-\alpha)\%$ PI for a future observation, y_f, has the form $[L_n, U_n]$ where $P(L_n \le y_f \le U_n)\xrightarrow{P}(1-\alpha)\%$ as the sample size $n\to\infty$. L_n and U_n are the lower and upper bounds of the PI respectively. Also $(1-\alpha)\%$ is the prescribed confidence level associated with PIs.

For the case of neural networks (NNs), several methods have been proposed in the literature for quantification of associated uncertainties [15–20]. PI construction methods can be broadly divided into parametric and non-parametric methods. In the former group, methods usually assume that the quantitative uncertainty (noise) is independently and identically distributed and follow a known distribution with unknown parameters. They then try to optimally estimate parameters of this distribution using available data. Once estimated, they use the distribution to construct intervals for model forecasts. The point is that data in many real world problems does not follow a priori known distribution. So application of parametric methods for construction of PIs in those cases is questionable. Non-parametric methods construct PIs without making any special assumption about the data and noise distributions. So, they have more flexibility and are theoretically more appropriate than parametric methods for practical applications.

The focus of this paper is on developing a framework for construction of PIs using IT2 Takagi–Sugeno–Kang FLSs (IT2 TSK FLSs). IT2 TSK FLSs have been widely used for system identification [21], control [22], and filter design [23]. This research is inspired by the fact the type reduced set carries valuable information about spread of the targets estimated by an IT2 TSK FLS model. This spread can provide valuable information about the level of uncertainty affecting estimations and forecasts. If wide (large spread), there is a high level of uncertainty associated with forecasts and they should be used with care. If narrow (small spread), forecasts are reliable and can be used confidently. Once the type-reduced set is defuzzified, this information is lost. Therefore, the proposed method here for construction of PIs using IT2 TSK FLSs does not require the defuzzification stage.

The major contribution of this paper is the introduction of a novel non-parametric method for construction of PIs using IT2 TSK FLSs. In contrast to parametric methods, the proposed method requires no special assumption regarding the data and its distribution to construct PIs. It also does not require calculation of complicate matrices and derivatives required by traditional PI construction methods. It is possible to build an empirical distribution for data by constructing PIs for different confidence levels ranging between zero to one. As a distribution-free (non-parametric) method, the IT2 TSK FLS-based method for construction of PIs is robust against violations of the normality assumption and offers a promising method for rapid construction of reliable PIs.

The proposed method is applied to construct PIs for seven synthetic and real-world benchmark case studies. The level of uncertainty in the synthetic data sets is controlled by adding homogeneous and heterogeneous noise. It is demonstrated that the proposed method generates high quality PIs, which are theoretically valid and practically informative. Also, the performance of the proposed method is stable in different replicates of experiments, which indicates it effectively and efficiently handles prevailing uncertainties in data. We also compare performance of the proposed method with two traditional NN-based methods for construction of PIs.

The rest of this article is organized as follows: Section 2 introduces IT2 TSK FLSs. PI assessment measures and indices are briefly discussed in Section 3. Section 4 describes the proposed method for PI construction using IT2 TSK FLSs. Simulation results are demonstrated in Section 5. Finally, we draw conclusions in Section 6.

2. Interval type-2 TSK fuzzy logic systems

The lth rule of an IT2 TSK FLS having p inputs, $x_1 \in X_1, \ldots, x_p \in X_p$, and one output, $y \in Y$, is expressed as [3,5],

$$R^l: \text{If } x_1 \text{ is } \tilde{F}_1^l,\ x_2 \text{ is } \tilde{F}_2^l, \ldots, \text{ and } x_p \text{ is } \tilde{F}_p^l, \text{ then}$$
$$y^l = C_0^l + C_l^1 x_1 + \cdots + C_p^l x_p \tag{1}$$

where $l=1, \ldots, M$, and M is the number of rules. $\tilde{F}_i^l$ is the ith IT2 FS ($i=1, \ldots, p$) composed of a lower and upper bound MF,

$$\mu_{\tilde{F}_i^l}(x_i) = [\underline{\mu}_{\tilde{F}_i^l}(x_i),\ \overline{\mu}_{\tilde{F}_i^l}(x_i)] \tag{2}$$

C_i^l is also an interval set, where its center and spread are c_i^l and s_i^l respectively,

$$C_i^l = [c_i^l - s_i^l,\ c_i^l + s_i^l] \tag{3}$$

where $i=0, \ldots, p$. C_i^l are the consequent parameters of the IT2 TSK FLS model. Given an input $x=(x_1, x_2, \ldots, x_p)$, the result of the input and antecedent operations (firing strength) is an interval type-1 set, $F^l = [\underline{f}^l, \overline{f}^l]$,

$$\underline{f}^l(x) = \underline{\mu}_{\tilde{F}_1^l}(x_1) * \underline{\mu}_{\tilde{F}_2^l}(x_2) * \cdots * \underline{\mu}_{\tilde{F}_p^l}(x_p) \tag{4}$$

$$\overline{f}^l(x) = \overline{\mu}_{\tilde{F}_1^l}(x_1) * \overline{\mu}_{\tilde{F}_2^l}(x_2) * \cdots * \overline{\mu}_{\tilde{F}_p^l}(x_p) \tag{5}$$

where * represents a t-norm. It is assumed that the singleton fuzzifier is used in obtaining (4) and (5).

y^l in (1) is the output from the lth If-Then rule, which is a T1 FS, $y^l = [y_l^l, y_r^l]$, and is evaluated as,

$$y^l = [y_l^l, y_r^l] = \left[\sum_{i=0}^{p} c_i^l x_i - \sum_{i=0}^{p} s_i^l |x_i|,\ \sum_{i=0}^{p} c_i^l x_i + \sum_{i=0}^{p} s_i^l |x_i|\right] \tag{6}$$

where $x_0 \equiv 1$. The final output of the IT2 TSK FLS model is obtained through combining the outcomes of M rules,

$$y = [y_l, y_r] = \int_{y^1 \in [y_l^1, y_r^1]} \cdots \int_{y^M \in [y_l^M, y_r^M]} \int_{f^1 \in [f_l^1, f_r^1]} \cdots \int_{f^M \in [f_l^M, f_r^M]} \frac{1}{\sum_{l=1}^{M} f^l y^l / \sum_{l=1}^{M} f^l} \tag{7}$$

The type-2 fuzzy output is then processed by the type-reduction operation, which combines the output sets and performs a centroid calculation that leads to a type-1 fuzzy set. y_l and y_r in (7) can be calculated using the iterative Karnik–Mendel (KM) procedure [24]. The centroid lies between the two end points calculated using the KM method. According to the KM method, y_l and y_r are calculated as below,

$$y_l = \frac{\sum_{l=1}^{L} \overline{f}_i^l y_l^l + \sum_{l=L+1}^{M} \underline{f}_i^l y_l^l}{\sum_{l=1}^{L} \overline{f}_i^l + \sum_{l=L+1}^{M} \underline{f}_i^l} \tag{8}$$

$$y_r = \frac{\sum_{l=1}^{R} \underline{f}_i^l y_r^l + \sum_{l=R+1}^{M} \overline{f}_i^l y_r^l}{\sum_{l=1}^{R} \underline{f}_i^l + \sum_{l=R+1}^{M} \overline{f}_i^l} \tag{9}$$