Omega 41 (2013) 31–40

Contents lists available at SciVerse ScienceDirect

Omega

journal homepage: www.elsevier.com/locate/omega

Ensuring units invariant slack selection in radial data envelopment analysis models, and incorporating slacks into an overall efficiency score

Barak Edelstein*, Joseph C. Paradi

Centre for Management of Technology and Entrepreneurship, Faculty of Applied Science and Engineering, University of Toronto, 200 College Street, Toronto, Ontario, Canada M5S 3E5

ARTICLE INFO

Available online 4 February 2012

Keywords:
DEA
Efficiency
LP
Banking
Risk
Optimization

ABSTRACT

This paper introduces a new methodology ensuring units invariant slack selection in radial DEA models and incorporating the slacks into an overall efficiency score. The CCR and BCC models are units invariant in their radial component, but not in their slack component, thus changing the units of measurement of one or more variables can change the models' solution. The proposed Full Proportional Slack (FPS) methodology improves the slack selections of the CCR and BCC models by producing slack selections that (a) are units invariant, thus producing fully units invariant models, (b) maximize the relative improvements represented by the slacks, and not their values, and (c) measure the full slacks that need to be removed from their corresponding variables. The FPS methodology is a fully oriented methodology first maximizing the improvements in the variables on the side of the orientation of the model. The Proportional Slack Adjusted (PSA) methodology incorporates the FPS slacks into an overall efficiency score, making it easier to interpret and use the results. The FPS and PSA methodologies are illustrated using an input oriented VRS Loan Quality DEA model with data from the retail branch network of one of Canada's largest banks.

1. Introduction

In 1957 Farrell [1] published his seminal paper "The measurement of productive efficiency". In time Farrell's work led to the development of Data Envelopment Analysis (DEA) in the seminal 1978 paper "Measuring the efficiency of decision making units" by Charnes et al. [2], which introduced the Constant Returns to Scale (CRS) DEA model, known as the CCR model. The Variable Returns to Scale (VRS) version of the CCR model, known as the BCC model, was next introduced in 1984 by Banker et al. [3]. DEA is a non-parametric Linear Programming (LP) methodology that defines a convex piecewise linear efficient frontier composed of the best performing Decision Making Units (DMUs), and calculates relative efficiency scores for the inefficient DMUs by measuring their relative distance to the efficient frontier. The CCR and BCC models are often solved in two stages [4]. In the first stage the radial improvement (θ) in the variables on the side of the orientation of the model is maximized, or in other words, the same proportional improvement is applied to all of the variables. The slacks measured in the second stage of the CCR and BCC models represent any remaining improvements measured along the skirt of the efficient frontier that are still possible after the proportional radial efficiency improvement has been applied to the variables on the side of the orientation of the model. The objective function of the second stage of the CCR and BCC models maximizes the sum of the input and output slacks simultaneously in order to identify the greatest total slack improvements possible after the first stage radial improvement was applied [4].

Two issues with the CCR and BCC radial models' slack selections have been identified in the literature. Lovell and Pastor [5] noted that while the CCR and BCC radial improvement measures are units invariant, their slacks are not. Units invariance is a desirable characteristic as it means that the model's solution is independent of the units of measurement of the model's variables. Charnes et al. [6] were the first to introduce a units invariant DEA model belonging to the Multiplicative group of DEA models. Lovell and Pastor [5] introduced a units invariant modified BCC model where in the second stage the slacks are weighted by the inverse of their corresponding variables' standard deviation. Sueyoshi et al. [7] and Cooper et al. [8] introduced models that are units invariant in their projections and efficiency scores. Tone's [9] slacks-based measure (SBM) model also produces meaningful units invariant scores, but employs only non-radial projections. Further discussion of the SMB model and its

Abbreviations: FPS, Full proportional slack; PSA, Proportional slack adjusted

* Corresponding author. Present address: 307-10 Navy Wharf Court, Toronto, Ontario, M5V 3V2 Canada. Tel.: +1 416 616 3266; fax: +1 416 766 8196.

E-mail addresses: barak.edelstein@alumni.utoronto.ca (B. Edelstein), paradi@mie.utoronto.ca (J.C. Paradi).

0305-0483/$ - see front matter © 2012 Elsevier Ltd. All rights reserved.
doi:10.1016/j.omega.2011.09.008

importance, relevance, and contribution to this paper, is provided in Section 4.2.

The literature includes a variety of additional units invariant DEA models and applications, many based on the SBM model and its variants. Asmild and Pastor [10] introduced an efficiency measure that is both units and translation invariant. Paradi et al. [11] used a second stage modified output-oriented SBM model to produce a composite performance index for bank branches using the efficiency scores of production, profitability, and intermediation efficiency evaluations as outputs with unity as an input. Avkiran [12] employed a non-oriented SBM model in the first of a 4 stage approach that utilizes a Tobit regression to study the influence of interest rates on bank efficiency. Tone [13] presented the dynamic SBM (DSBM) model to evaluate full and period specific efficiencies while accounting for carry-over activities between time periods. Avkiran [14] applied SBM to examine the profit efficiency of commercial banks, and a non-oriented network SBM [15] approach using simulated profit center data. Yu [16] applied a network SBM model in evaluating airport performance. Premachandra et al. [17] utilized an additive super-efficiency DEA model to predict corporate failure and success. Barros et al. [18] applied the weighted Russell directional distance model [19] to assess a bank's technical efficiency. Avkiran and Morita [20] examined bank performance from a multi-stakeholder perspective using the range adjusted measure (RAM) [8].

A second CCR and BCC slack related issue addressed in the literature by Sueyoshi et al. [7] and Cooper et al. [8] is that the first stage efficiency score of the CCR and BCC models does not incorporate the slacks into it and thus does not capture the full inefficiency of a DMU, instead reporting it in two separate components. Sueyoshi et al. and Cooper et al. introduced models that incorporate the slacks into units invariant efficiency scores while also producing units invariant projections and reference sets. Sueyoshi et al. [7] introduced the slack-adjusted (SA-DEA) model that incorporates the slacks into an overall efficiency score by subtracting from the efficiency score θ the average of the ratios of slack to the maximal variable value. Cooper et al. [8] introduced the RAM, a methodology related to the Additive model, but also suggested three relevant models that integrated slack ratios into the input oriented CCR and BCC radial DEA models. The first related model, which does not use the RAM approach, calculates θ and the slacks in the objective function by minimizing θ and subtracting from it (i.e., maximizing) the product of ε times the sum of the averaged ratios of the input slacks to the corresponding input variable values and the averaged ratios of the output slacks to the corresponding output variable values [8]. The second related model integrates the RAM approach with the BCC model to calculate θ and the slacks by minimizing θ and subtracting from it the product of ε times the sum of the ratios of the slacks to the range in the corresponding variable values [8]. The third relevant model incorporates the slacks into an overall efficiency score by subtracting the average of the ratios of the slacks to the range in the corresponding variable values from the efficiency score θ [8]. The Sueyoshi et al. and Cooper et al. approaches made important contributions to the DEA literature, and this paper builds on those contributions and suggests an alternative approach to dealing with several issues that remain unresolved in those models.

The ratios used in the Sueyoshi et al. and Cooper et al. models do not capture the true relative inefficiency represented by the slacks. As with the CCR and BCC models, the slacks in the variables on the side of the orientation of these models are measured along the efficient frontier after the proportional radial efficiency improvements have been applied and thus do not measure the actual slack amount that needs to be removed from the entire variable. Also, the slacks in the SA-DEA and RAM models are also usually divided by a value different than the DMU's own value for that variable as the maximal and minimal variable values, and thus the range in the variable values, are dependent on the dataset. The inclusion or exclusion of certain DMUs, especially ones which are potentially outliers, will alter the overall efficiency scores calculated by these models. These approaches to integrating the slacks into overall efficiency scores can also produce negative overall efficiency scores as the averaged slack ratios that are subtracted from θ could be greater than θ.

A second group of models that accounts for the full inefficiency in the data exists, but these are based on non-radial projections while the focus of this paper is slack selection in the CCR and BCC radial DEA models. The Additive model on which this group of models is based was introduced by Charnes et al. [21] in 1985, and provides a non-oriented efficiency measure that simultaneously reduces the inputs and increases the outputs by only taking the slacks into account when measuring efficiency. The Additive model, however, is not units invariant. The RAM model farther developed the Additive model, exhibiting favorable characteristics such as units and translation invariance [8]. The SBM model expanded on the Additive model by also employing non-radial, non-oriented projections, but the SBM model is units invariant and produces meaningful efficiency scores [9].

2. Units invariance and slack selection in radial DEA models

2.1. Units invariance

Units invariant DEA models produce optimal solutions that are independent of the units of measurement of the model's variables. The models proposed in this paper are units invariance in terms of the optimal solution to the objective functions, the relative slack improvements, the efficiency scores, the reference sets, and the projections that the models produce in terms of λ values. In other words, these measures will not change should the units of measurements of any variable be changed through multiplication by some positive factor. Note, however, that the slack values produced by the models proposed here are reported in terms of the same units of measurement as their corresponding variables. Should the units of measurement of any variable change through multiplication by some positive factor, the slack values will change by the same multiplicative factor.

2.2. Slack selection issues with the CCR and BCC models

This paper deals with four issues in the slack selection approach of the CCR and BCC models:

(a) While the CCR and BCC radial improvement calculations are units invariant, these models are not units invariant in their slack component and changing the units of measurement of one or more variables through multiplication by any factor can change the model's solution [5].
(b) The second stage maximization of the sum of the slacks maximizes the total value of the slacks and not the total relative (percentage) improvements that the slacks represent.
(c) The slacks in the variables on the side of the orientation of the model are measured along the skirt of the frontier after the radial improvement found in the first stage of the model has been applied, thus favouring output slack expansion when simultaneously maximizing the sum of the input and output slacks by underestimating the input slacks in the case of an input oriented model, or overestimating the output slacks in the case of an output oriented model.