Decision Support

Operations research models for coalition structure in collaborative logistics

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ABSTRACT

Given a set of players and the cost of each possible coalition, the question we address is which coalitions should be formed. We formulate mixed integer linear programming models for this problem, considering core stability and strong equilibrium. The objective function looks for minimizing the total cost allocated among the players. Concerned about the difficulties of managing large coalitions in practice, we also study the effect of a maximum cardinality constraint per coalition. We test the models in two applications. One is in collaborative forest transportation and the other one in inventory of spare parts for oil operations. In these situations, collaboration opportunities involving significant savings exist, but for several reasons, it may be better to group the players in different sub-coalitions rather than in the grand coalition. The models we propose are thus relevant for deciding how to partition the set of players. We also prove that if the strong equilibrium model is feasible, its optimal cost is equal to the optimal cost of the core stability model and, consequently, a coalition structure that solves one problem also solves the other problem. We present results that illustrate this property. We also present results where the core stability problem is feasible and the strong equilibrium problem is infeasible. Setting an upper bound on the maximum cardinality of the coalitions, allows us to study the marginal savings of enlarging the cardinality of the coalitions. We find that the marginal savings of allowing one more player significantly decreases as the bound increases.

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1. Introduction

Collaboration among different agents is an effective way to improve logistic operations. Evidence has been provided in many industries. For instance, Nagarajan and Sŏsić (2009) and Nagarajan and Bassok (2008) refer to a variety of contexts where different suppliers create coalitions to achieve the benefits of collaboration, including complementary component-suppliers for car assembly systems, the health-care industry and various service sectors. Other examples arise in inventory management (Chen, 2009; Özen, Fransoo, Norde, & Slikker, 2008) and transport (Frisk, Göthe-Lundgren, Jörnsten, & Rönnqvist, 2010; Lozano, Moreno, Adenso-Díaz, & Algaba, 2013). In order to implement collaboration, there needs to be an agreement among the agents with respect to how to share the benefits. A growing body of literature in Operations Research and Management Science has applied principles of cooperative game theory to resolve this sharing issue. One of the main streams in this literature aims at defining allocation rules

to predict or prescribe how rational players will distribute the gains they obtain from cooperation, assuming in general that the grand coalition will be formed, in line with the classical approach to transferable utility games (Meca-Martínez, Sánchez-Soriano, García-Jurando, & Tijs, 1998). The formation of the grand coalition may find support in the assumption of superadditive games, i.e., where the value of a coalition is at least as good as the sum of the values of its members acting separately. This would mean that the more agents collaborate, the better is the outcome they achieve (or at least "not worse" than if only some of them or none of them collaborate). In practice, however, several reasons might exist for the grand coalition not to be formed. Some of these reasons, for example, are the managerial complexity of conforming large coalitions and the political issues that can affect the decision of changing from a non-collaborative to a collaborative solution. For example, in transport operations, collaboration usually involves just a few partners, because as the number of partners grows coordinating the cooperation becomes more problematic and/or costly (Lozano et al., 2013). Then, given a set of players and the cost of each possible coalition, the relevant question arises of which coalitions should be formed. The formed coalitions define a partition of the set of players, which in game theory is called a *coalition*

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structure (Aumann & Dreze, 1974). In contrast to a vast body of game theory literature dealing with coalition structure and coalition formation issues (see e.g. Greenberg, 1994), the OR/MS literature is more sparse in this stream. Some of the exceptions addressing issues related to coalition structures in OR are Axelrod, Mitchell, Thomas, Bennett, and Bruderer (1995), Leng and Parlar (2009), Nagarajan and Sösic (2009) and Sösic (2006, 2010). Other recent works have acknowledged the relevance of the coalition structure issues, although without addressing them (Chen, 2009; Kim & Jeon, 2009; Lozano et al., 2013).

In this article, we propose mixed integer linear programming formulations to simultaneously approach the coalition structure and cost allocation problems. Our formulations aim at deciding which coalitions should be formed and the cost allocation to the players in each of them, subject to stability conditions. These stability conditions are of two types. First, we formulate constraints to model internal stability, i.e., the core constraints for the members within a coalition. Second, we formulate constraints to model the strong stability, i.e., where no group of players, whether from the same coalition or from different ones, can get together and form new coalition(s) in such a way that they are all better off (Aumann, 1959; Hart & Kurz, 1983). Concerned about the complications of implementing large coalitions in practical situations, we also incorporate conditions on the maximum number of players that can form a coalition. We test our models in two applications. The first is a collaborative forest transportation problem presented by Frisk et al. (2010), which includes eight companies operating in Sweden. As the companies operate in similar regions, the opportunities for collaborative transportation represent important savings, in the range 5–15%. The second is a problem on collaboration in inventory of spare parts for oil operations, where risk pooling represents an important source of savings of about 20% of the inventory costs (Guajardo, Rönnqvist, Halvorsen, & Kallevik, 2014).

Our work contributes in both practical and theoretical aspects. In the practical aspects, we approach two cases motivated in forestry and oil operations, where collaboration in transport and inventory operations improve the non-cooperative case. Since we do not assume any particular form of the cost functions, the potential for applications may also be extended to other industries using collaborative logistics. In the theoretic aspects, we introduce model formulations for a relevant problem in the interface of Game Theory and Operations Research. The models consist of a set partitioning problem subject to stability constraints. Because of the combinatorial structure of this problem and its relevance in theory and practice, we believe our work opens a rich source of challenges where OR solution methods can play an important role.

The remainder of this article is organized as follows. In Section 2, we summarize concepts of stability in cooperative games. In Section 3, we formulate models for coalition structure and cost allocation. In Section 4, we apply the models in collaborative problems in the forestry and oil industries, and also discuss its applicability in larger scale problems. Our conclusions are presented in Section 5.

2. Stability in cooperative games

A growing body of literature has incorporated concepts from cooperative game theory in collaborative logistics. Fiestras-Janeiro, García-Jurado, Meca, and Mosquera (2011) review a number of references on collaborative inventory management. In other areas of operations management, however, the use of cooperative game theory is much less prevalent (Hu, Caldentey, & Vulcano, 2013). In the following, we introduce some notation and summarize the concepts on stability in cooperative games that are relevant for our article.

We consider a game consisting of a set of players N. We refer to the cardinality of this set by $n = |N|$. Players can form coalitions, in order to take advantage of collaboration opportunities. The set N is the *grand coalition*. We refer by C_Z to the cost incurred by coalition Z, where $Z \subseteq N$. We assume $C_Z \geqslant 0$ for all Z. The cost function C is called the *characteristic function* of the game. We refer by u_j to the cost allocated to player j, for each player $j \in N$. We assume $u_j \geqslant 0$ for all j. A cost allocation vector $u = (u_1, u_2, \ldots, u_n)$ is said to be in the *core* of the game (Gillies, 1959) if it satisfies the following constraints:

$$\sum_{j \in Z} u_j \leqslant C_Z \quad \forall Z \subset N \tag{1}$$

$$\sum_{j \in N} u_j = C_N \tag{2}$$

Constraints (1) correspond to the rationality conditions, which state that there is no subset Z of players such that should they form a coalition separately from the rest they would perceive less total cost than the total cost allocated to them in u. In the case of Z containing only one player, the constraint corresponds to the individually rational condition, which states that the cost allocated to each player j must not be greater than its stand-alone cost. Constraint (2) corresponds to the efficiency condition, which states that the sum of the costs allocated to all the players must be equal to the optimal expected cost of the grand coalition. The core of the game is the set of all vectors u satisfying constraints (1) and (2). An allocation that belongs to the core is said to be *stable*. In order to differentiate this type of stability from a following concept, we will refer to it as *core stability*.

As mentioned in the introduction, several reasons might exist for the grand coalition not to be formed. Therefore, it becomes relevant to study the stability of cooperative situations where different group of players may form different coalitions. The formed coalitions define what in game theory is called a *coalition structure* (Aumann & Dreze, 1974). Following the notation of Sösic (2006), a coalition structure £ is a partition on N, that is, £ $= \{Z_1, \ldots, Z_r\}$, $\bigcup_{i=1}^{r} Z_i = N, Z_i \cap Z_j = \emptyset, i \neq j$.

We assume the cost of all coalitions are given parameters. This in practice does not necessarily mean that the costs and the information needed to compute them are known by all the players, but it is enough if there would be a decision maker who could gather this information. Then, this decision maker needs to find the coalitions that should be formed, aiming at minimizing the total allocated cost while satisfying the constraints. In the two cases that primarily motivated our work, this decision maker emerges in two different ways. In the forestry case, there is a consulting team that gathers the information from the companies and conduces the analysis, in order to come up with a suggestion on how to implement the collaboration among the companies. It is crucial that this third party manages the information under confidentiality, since part of the information is sensitive to the companies. In fact, this has been the only viable way for them to provide the information. The third party also needs to be impartial and, therefore, the minimization of total cost is an appropriate overall objective when making the suggestion on how to implement the collaboration. This objective is also convenient from an environmental point of view, as usually the savings in transportation cost are associated to a reduction of emissions from the trucks. In the oil case, the suggestion is made by a main company which has the operational responsibility for all the warehouses that hold inventory of spare parts, despite some different ownership structures involving other companies and stakeholders. The data from all these warehouses are managed in the same enterprise resource planning system SAP. The operational responsible acts with impartiality and a cost