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Approaches to group decision making with incomplete information based on power geometric operators and triangular fuzzy AHP

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ABSTRACT

In this paper, we investigate the multiple criteria group decision making (MCGDM) problems in which decision makers (DMs)' preferences on alternatives (criteria) are depicted by triangular fuzzy numbers and take the form of incomplete reciprocal comparison matrices. We aim to develop integrated methodologies for the MCGDM problems. First of all, we develop a triangular fuzzy power geometric (TFPG) operator and a triangular fuzzy weighted power geometric (TFWPG) operator for aggregating the DMs' preferences into the group preferences. Furthermore, we construct a consistent recovery method and a δ-consistent recovery method for estimating the missing preferences. Next, we propose two integrated approaches to the aforementioned MCGDM problems by utilizing triangular fuzzy analytic hierarchy process (TFAHP) to combine the TFPG (TFWPG) operator, the recovery methods and extent analysis method (EAM) effectively. Finally, an illustrative example of small hydropower (SHP) investment projects selection is given to show our approaches.

1. Introduction

Since Analytic Hierarchy Process (AHP) as a priority theory was originally presented by Saaty (1977, 1980), it has caught much attention of researchers (Aczel & Saaty, 1983; Aguarn & Moreno-Jimnez, 2000; Aguarn, Escobar, & Moreno-Jimnez, 2003; Aguarn & Moreno-Jimnez, 2003; Escobar, Aguarn, & Moreno-Jimnez, 2004; Forman & Peniwati, 1998; Ramanathan & Ganesh, 1994; Xu, 2000). So far, Saaty's AHP (S-AHP) has become one of the most common tools applied to multiple criteria decision making (Saaty, 1980; Aczel & Saaty, 1983; Davies, 1994; Escobar et al., 2004; Forman & Peniwati, 1998; Ramanathan & Ganesh, 1994). In a real multiple criteria decision making (MCDM) problem based on the AHP, there generally exist vagueness and incompleteness in the preference information (or reciprocal comparisons) on alternatives (criteria), which are given by the decision maker (DM) because of his/her limited knowledge and experience on the alternatives (criteria) estimated and evaluated. Zadeh (1965) proposed Fuzzy Sets (FSs) theory for dealing with vague information. Triangular fuzzy number and trapezoidal fuzzy number are two restrict fuzzy sets with convexity and normalization, and have been widely and successfully applied to modeling DMs' preferences. Naturally, many researchers have combined triangular (trapezoidal) fuzzy number with S-AHP to develop various triangular (trapezoidal) fuzzy AHP (TFAHP) methodologies. Such as, Van Laarhoven and Pedrycz (1983) presented a TFAHP methodology for a MCDM problem in which triangular fuzzy numbers were utilized to depict the DM's preferences on alternatives (criteria) in the form of reciprocal comparison matrices. They defined some basic operations of triangular fuzzy numbers and developed logarithmic least-square method (LLSM) for obtaining priorities of the alternatives (criteria). Buckley (1985) developed a TFAHP methodology for a MCDM problem in which trapezoidal fuzzy numbers were used to express the DM's preferences on alternatives (criteria) in the form of reciprocal comparison matrices. He derived the priorities of the alternatives (criteria) by using the operations of trapezoidal fuzzy numbers and geometric mean (GM). In addition, Csutora and Buckley (2001) introduced triangular fuzzy number to the Lambda-Max method proposed by Saaty (1980) for obtaining the priority vector of alternatives (criteria) in S-AHP. To overcome the computation complexity of the method in Van Laarhoven and Pedrycz (1983), Chang (1996) presented an extent analysis method (EAM) for multiple criteria group decision making (MCGDM) problems in which decision makers (DMs)' preferences on alternatives (criteria) were depicted by triangular fuzzy numbers and took the

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form of incomplete reciprocal comparison matrices. In Chang (1996), an synthetic extent formula was developed firstly; next,an possibility degree to which one triangular fuzzy number was larger than or equal to another was defined; and then, an overall possibility degree to which one of a set of triangular fuzzy numbers was larger than or equal to the rest was also defined; on the basis of the synthetic extent formula, possibility degree and overall possibility degree, the EAM was developed for obtaining the priority of alternatives (criteria) in the MCGDM problems. Since the Chang's method, by its very nature, is a simple row mean which is relatively easier than the other priority methods of TFAHP (Xu & Liao, 2014), it has been applied to many different areas, such as petroleum exploitation (Zhu, Jing, & Chang, 1999), new production development (Bykozkam & Feyzioglu, 2004), capital investment (Tang & Beynon, 2005), ERP system selection (Cebeci, 2009), intelligent system (Chang, Wu, & Lin, 2009), knowledge management (Chang & Wang, 2009), personnel management (Gngor, Serhadlioglu, & Kesen, 2009), supply chain management (Wang, Cheng, & Cheng, 2009; Jakhar & Barua, 2014), power station location selection (Kabir & Sumi, 2014), and construction project management (Taylan, Bafail, Abdulaal, & Kabli, 2014).

Generally, a typical TFAHP based MCGDM process, in which the DMs' preferences on alternatives (criteria) are depicted by triangular fuzzy numbers and take the form of incomplete reciprocal comparison matrices where some preferences are missing, involves five basic steps including (1) construction of hierarchy structure by decomposing general decision objective into criteria and forming alternatives with respect to all the criteria, (2) construction of reciprocal comparison matrices of alternatives (criteria), where the entries are incomplete triangular fuzzy preferences (i.e. there exist missing preferences) given by the DMs on the alternatives (criteria), (3) completion of reciprocal comparison matrices with missing preferences, (4) aggregation of all DMs' preferences on alternatives (criteria) into group preferences, and (5) derivation of priorities of the alternatives with respect to the general objective. Furthermore, the supports for each DM from others should be considered in aggregation process on the basis of power average (PA) operator developed by Yager (2001) and the extreme preferences should be leveled rationally (Mikhailov, 2003; Xu & Yager, 2010). Existing methods provide partial solutions to all of the MCGDM problems. For example, Chang (1996) did not take into account the completion of missing preferences and the supports. Zhu et al. (1999) verified the possibility degree in Chang (1996) and improved it. Furthermore, Wang, Luo, and Hua (2008) examined Chang's EAM (C-EAM) through numerical examples and pointed that the synthetic extent formula is incorrect. Xu and Yager (2010) considered the support and extreme preferences in the form of interval values instead of triangular fuzzy numbers, and he did not focus on the incompleteness of fuzzy preference relations. Liu, Zhang, and Zhang (2014) only investigated consistency of triangular fuzzy reciprocal preference relations. Abdullah and Zulkifli (2015) presented a method of integrating interval type-2 trapezoidal fuzzy numbers, AHP and decision making trial and evaluation laboratory (DEMATEL) considering casual relations between criteria for fuzzy MCDM. He did not take into account group decision context and incompleteness of trapezoidal fuzzy preference relations. Zhu et al. (1999) presented a completion method for obtaining the missing preferences without consideration of the other problems. Wang and Chen (2014) proposed a logarithmic least squares model to estimate missing values for incomplete interval fuzzy preference relations without considering group decision context. Meng and Chen (2015) developed a goal programming model for obtaining the missing values of incomplete fuzzy preference relation depicted by membership degree function with single value in group decision making. He, however, did not take into account the support and extreme preferences situation. Additionally, he took single membership degree value rather than fuzzy numbers to depict the DMs' preferences.

Obviously, There is an absence of synthetic methodologies to deal with the aforementioned MCGDM problems. The main aim of this paper is to develop such integrated approaches which will be helpful to improve the capacity of Expert and Intelligent Systems. To do this, the remainder of the paper is organized as follows. Section 2 reviews the concepts of triangular fuzzy number, PA operator, and Chang's possibility degree. In Section 3, we develop a triangular fuzzy power geometric (TFPG) operator and a triangular fuzzy weighted power geometric (TFWPG) operator, study some of their properties, such as commutativity, idempotency, boundedness and discuss the relationship between the TFPG and TFWPG operators. Section 4 proposes a consistent recovery method and a δ-consistent recovery method for estimating the missing preferences. In Section 5, we develop two integrated approaches to the aforementioned MCGDM problems by using TFAHP to combine the TFPG (TFWPG) operator, recovery methods, C-EAM. Section 6 introduces a practical example of small hydropower (SHP) investment project selection. Finally, some concluding remarks and further research are provided in Section 7.

2. Preliminaries

In this section, we introduce the basic concepts of triangular fuzzy number and its operation laws, distance measure of triangular fuzzy numbers, power average geometric operators, and possibility degree of comparing two triangular fuzzy numbers.

2.1. Triangular fuzzy numbers and their basic operations

Definition 1. Let $F(R)$ be the complete set of all fuzzy sets in real line R. Given a set $\widetilde{A} \in F(R)$, the support and height of $\widetilde{A}$ can be defined as follows, respectively:

(1) $Supp(\widetilde{A}) = \{x | \mu_{\widetilde{A}}(x) > 0, x \in R\}$,
(2) $Hgt(\widetilde{A}) = \max_{x \in R}\{\mu_{\widetilde{A}}(x)\}$,

where $\mu_{\widetilde{A}}(x)$ is referred to as the membership function characterizing the fuzzy set A in R and associates with each point in R a real number in the interval [0,1] (Zadeh, 1965).

Definition 2 Zadeh (1965). A fuzzy set $\widetilde{A} \in F(R)$ is convex if and only if

$$\mu_{\widetilde{A}}(\lambda x_1 + (1-\lambda)x_2) \geqslant \min\{\mu_{\widetilde{A}}(x_1), \mu_{\widetilde{A}}(x_2)\},$$

for all x_1 and x_2 in $Supp(\widetilde{A})$ and all λ in [0, 1].

Definition 3. A fuzzy set $\widetilde{A} \in F(R)$ is normalized if $Hgt(\widetilde{A}) = 1$.

Definition 4 Zimmermann (1991). A fuzzy set $\widetilde{A} \in F(R)$ is a fuzzy number if it is convex and normalized.

Definition 5 Chang (1996). A fuzzy number $\widetilde{A}$ is a triangular fuzzy number (TFN) if its membership degree function is expressed mathematically as follows:

$$\mu_{\widetilde{A}}(x) = \begin{cases} \frac{x-l}{m-l}, & \text{if } l \leqslant x \leqslant m; \\ \frac{u-x}{u-m}, & \text{if } m \leqslant x \leqslant u; \\ 0, & \text{otherwise} \end{cases} \qquad (1)$$