1st International Conference 'Economic Scientific Research - Theoretical, Empirical and Practical Approaches', ESPERA 2013

MADM in the case of simultaneous equations models and economic applications

Daniel Ciuiu[a,b]*

[a]Technical University of Civil Engineering Bucharest, Bd. Lacul Tei No. 124, Bucharest, 020396, Romania
[b] Institute for Economic Forecasting, Calea 13 Septembrie No. 13, Bucharest 050711, Romania

Abstract

In this paper we will solve MADM (Multiple Attribute Decision Making) problems in the case of the simultaneous equations models. The p dependent variables are considered the stochastic criteria, and the alternative decisions are given by points in R^k, where k is the number of explanatory variables. The set of alternative decisions contains the points from the dataset from which we estimate the regression coefficients, but it is completed by simulation on an interval in R^k. The senses are given by the economic interpretation of the variables $Y_i, i = \overline{1,p}$. We consider as economic application the GDP/capita and long term unemployment rate in terms of computer skills

Keywords: MADM; multi-linear regression; simultaneous equations; GDP/capita; computer skills; long term unemployment rate;

1. Introduction

A MADM (Multiple Attribute Decision Making) can be formulated as follows (Văduva and Resteanu, 2009; Văduva, 2012). There are m decision alternatives to be taken and n criteria or attributes used to determine the best (optimum) alternative decision. To select the best decision according these attributes there is defined for each

---

* Corresponding author. Tel.:+0040216836152.
 *E-mail address:*dciuiu@yahoo.com

attribute the sense: minimum, if the attribute is a loss, respectively maximum if it is a gain. Because in general there exist no decision optimal with respect each criterion, we need an importance vector, P, that express the importance given for each criterion. Usually P is a probability vector.

The data of a MADM problem can be represented as in Table 1, where A_1, A_2,…, A_m are the decision alternatives, C_1, C_2,…, C_n are the criteria, the $m \times n$ matrix of a_{ij}, $1 \le i \le m$; $1 \le j \le n$ is the matrix of entries (the values of criterion j if we take the decision i), P is the importance vector and *sense* is the sense vector.

Table 1: The MADM problem.

	C_1	C_2	**...**	C_n
A_1	a_{11}	a_{12}	...	a_{1n}
A_2	a_{21}	a_{22}	...	a_{2n}
...	...	...	...	**...**
A_m	a_{m1}	a_{m2}	...	a_{mn}
P	p_1	p_2	...	p_n
sense	*sense*$_1$	*sense*$_2$	**...**	*sense*$_n$

One of the steps in solving a MADM problem is to estimate the vector P of importance weights. The first method presented in Văduva (2012) is the method of eigenvector. We have given in this case the matrix B of relative importance: $B = \begin{vmatrix} 1 & \frac{p_1}{p_2} & \cdots & \frac{p_1}{p_n} \\ \frac{p_2}{p_1} & 1 & \cdots & \frac{p_2}{p_n} \\ \cdots & \cdots & \cdots & \cdots \\ \frac{p_n}{p_1} & \frac{p_n}{p_2} & \cdots & 1 \end{vmatrix}$. In the above formula $b_{ij} = \frac{p_i}{p_j}$ is given by the decedent, and it represents the relative importance of the criterion i with respect the criterion j. These values can be any positives ones, but, in the ideal case (when the decedent is not contradictory) there must be fulfilled some conditions (Văduva, 2012): $\begin{cases} b_{ji} = \frac{1}{b_{ij}} \\ b_{jk} = \frac{p_j}{p_k} \end{cases}$. We compute the maximum eigenvalue of B, $\lambda_{\max}$, and P is its corresponding eigenvector.

Another method to estimate P is the least squares method (Văduva, 2012). We have to solve the minimum problem, considering the above matrix B:
$$\begin{cases} \min \sum_{i=1}^{n} \sum_{j=1}^{n} \left(b_{ij} p_j - p_i \right)^2 \\ \sum_{j=1}^{n} p_j = 1 \end{cases}$$
, where $B = \left(b_{ij} \right)_{i,j=\overline{1,n}}$ is the above matrix of relative importance. Using the Lagrange multipliers' method we solve first a linear system of equations, obtaining $p_i = p_i(\lambda)$. Next we compute λ from $\sum_{j=1}^{n} p_j = 1$, and, using this value, we estimate p_i.

In the ideal case (when the decedent is not contradictory), the eigenvector value leads to $\lambda_{\max}$ lambda max=k, and the weights are proportional to the values of one row (all the rows of matrix are proportional). The same weights are obtained by the least squares method, and the minimum sum is zero.

Another step is to transform the entries a_{ij} such that all entries are of the same type, i.e. all fuzzy, or all cardinal. In this paper we consider the transformation such that all entries become cardinal. If the corresponding entries of the criterion j, a_{ij}, $i = \overline{1,m}$ are stochastic, characterized by the random variable X, we make two cardinal criteria: the first one is the expectation of the random entry, and second one is informational, as follows. The entry is normalized (by dividing to standard deviation), and, for second criterion, it is computed the Shannon entropy (Văduva and Resteanu, 2009; Văduva, 2012). The Shannon entropy can be replaced by Onicescu informational energy (Onicescu, 1966; Onicescu and Ştefănescu, 1979; Petrică and Ştefănescu, 1982). The first cardinal criterion made from the stochastic criterion j has the entry on the row I (corresponding to the decision i) equal to $E(X)$, and the second one has the entry on the same row either equal to the above Shannon entropy, either equal to the above Onicescu's informational energy. The sense of expectation is the same as for original criterion, while for the informational one,