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Comparison of some aggregation techniques using group analytic hierarchy process

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ABSTRACT

Group decision making is an important part of multiple criteria decision making and the analytic hierarchy process (AHP). The aim of this paper was to compare group AHP methods. Seven simple group AHP aggregation techniques that could be attractive for applications selected from the vast array of group AHP models proposed in the literature were selected for evaluation. We developed three new measures of evaluation: group Euclidean distance, group minimum violations, and distance between weights for the purpose of evaluation. The results of seven group AHP methods of the theoretical example were evaluated by three new evaluation measures, satisfactory index and fitting performance index. Furthermore, a case study of a decision making problem from the construction engineering field was performed and nine group AHP aggregation techniques, seven of them formerly presented and two new two stage group approaches were applied. Finally, the case study was evaluated using all five measures for each of the nine group decision making methods. The results showed that not all group AHP methods are equally convenient and that the selection of the method depended on the specific application.

1. Introduction

Group decision making is becoming an increasingly important part of multiple criteria decision making (Ahmad, Saman, Mohamad, Mohamad, & Awang, 2014; De Brucker, Macharis, & Verbeke, 2013; Ishizaka & Labib, 2011a; Kuzman, Grošelj, Ayrilmis, & Zbasnik-Senegacnik, 2013; Ren, Fedele, Mason, Manzardo, & Scipioni, 2013; Skorupski, 2014; Wang, Peng, Zhang, & Chen, 2014; Yu & Lai, 2011). Multiple stakeholders can contribute a variety of experiences, expertise and perspectives, and a group can better deal with the complexity of the problem than a single decision maker (DM). The analytic hierarchy process (AHP) (Saaty, 1980) is deemed to be one of the most appropriate methods for group multiple criteria decision making (Peniwati, 2007). In group AHP, four basic approaches for deriving the group priority vector from comparison matrices of DMs are suggested (Dyer & Forman, 1992; Ishizaka & Labib, 2011b; Lai, Wong, & Cheung, 2002). The group can try to reach a consensus on a meeting, first in developing the hierarchy and then in generating pairwise comparisons. If they cannot reach a consensus regarding a particular judgment, they can vote or try to achieve a compromise. Social choice theory with voting systems (Taylor & Pacelli, 2008) can be combined with AHP (Srdjevic, 2007). The aggregation of individual priorities (AIP) and the aggregation of individual judgments (AIJ) are two main mathematical aggregating methods (Forman & Peniwati, 1998). The most widely used aggregation technique is the weighted geometric mean method for AIJ (WGM–AIJ), which has been applied in numerous applications (Ananda & Herath, 2008; Cortés-Aldana, García-Melón, Fernández-de-Lucio, Aragonés-Beltrán, & Poveda-Bautista, 2009; de Luca, 2014; Lee, Chang, & Lin, 2009; Srdjevic, Lakicevic, & Srdjevic, 2013; Sun & Li, 2009).

The decision maker is satisfied if the final group priorities are as close as possible to his judgments, priorities or his ranking of criteria. Unlike the single DM case, in the group case there are not many studies comparing the results of different AHP group approaches, which results in a lack of measures for comparing group methods (Hosseinian, Navidi, & Hajfathaliha, 2012; Huang, Liao, & Lin, 2009).

The main objective of this paper is to develop new measures for evaluating AHP group methods. We proposed three new measures: group Euclidean distance (GED), group minimum violations (GMV), and distance between weights (WD). The second aim of the present study was to select the most appropriate group AHP method for employment in the applications. Although WGM–AIJ is the most often applied method it is not necessary the most suitable method.

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For the comparative study we selected WGM–AIP, weighted arithmetic mean method (WAM), and some recently presented models in addition to WGM–AIJ (Hosseinian et al., 2012; Huang et al., 2009; Regan, Colyvan, & Markovchick-Nicholls, 2006; Sun & Greenberg, 2006). These models were selected because they are easy to understand and could be attractive for many applications.

The three new measures, the satisfactory (SAT) index (Huang et al., 2009) and the fitting performance (FP) index (Hosseinian et al., 2012) were employed in the evaluation study, which compared seven group AHP methods in a theoretical example. Additionally, a case study that compared the criteria for selecting building construction method and material for an industrial type of building was performed. In the study, three groups of stakeholders were included in the decision making. To aggregate the stakeholders' judgments we suggest utilizing AIJ within the groups first and then AIP between the groups. In the paper we proposed two new stage group approaches, namely WGM–WAM and WGM–LW-AHP. Seven known group AHP methods and two newly proposed were applied in the case study for deriving group priorities. The results of nine group AHP methods were compared with five evaluation measures: GED, GMV, WD, SAT index, and FP index.

The next section offered a brief description of group AHP methods applied in the study. Further, we proposed the measures for evaluating the group AHP methods. The theoretical part of paper was followed by the theoretical example and a case study. Finally, some conclusions were provided.

2. Revision of group AHP prioritization methods

Let n be the number of criteria (or alternatives) and m the number of DMs. The standard AHP 1–9 scale (Saaty, 1980) was used for the judgments of each DM, which were written in the comparison matrices $\mathbf{A}^{(k)} = (a_{ij}^k)_{n\times n},\ k = 1,\ldots,m$. If the DMs' opinions were not equally important, the relative importance weight of kth DM's opinion was denoted by α_k, for $k = 1,\ldots,m$, with $\alpha_k > 0$ and $\sum_{k=1}^m \alpha_k = 1$.

There are many methods for deriving priority vectors but in this study we primarily used the eigenvector method (Saaty, 1980) resulting in $\mathbf{w}^k = (w_1^k,\ldots,w_n^k)^T,\ k = 1,\ldots,m$ as DMs' priority vectors. In the study we focused on the additive error structure $a_{ij} = \frac{w_i}{w_j} + \varepsilon_{ij}$ for inconsistent comparison matrix A and used additive normalization condition $\sum_{i=1}^n w_i = 1$ for all priority vectors for one or more DMs (Sun & Greenberg, 2006). The consistency of judgments in the comparison matrix A was measured by the consistency ratio $CR_A = \frac{\lambda_{A,\max}-n}{(n-1)RI_n}$, where RI_n was the average random consistency index. A consistency ratio of less than 0.1 was considered acceptable.

Of the AIJ methods WGM–AIJ is the only method that meets several required axiomatic conditions, such as the reciprocal property (Aczél & Alsina, 1986). The individual judgments $a_{ij}^k,\ k = 1,\ldots,m$ were aggregated into a group judgment a_{ij}^{WGMM} by weighted geometric mean:

$$a_{ij}^{WGMM} = \prod_{k=1}^m (a_{ij}^k)^{\alpha_k} \tag{1}$$

The group priority vector was derived from the group comparison matrix A^{WGMM} by the eigenvector method.

The AIP is a suitable method when a group is non-homogenous and consists of stakeholders from different fields. Both the WAM and WGM approaches can be used for the AIP. First, each DM k, $k = 1,\ldots,m$, applies for the eigenvector method for deriving the priority vector $\mathbf{w}^k = (w_1^k,\ldots,w_n^k)^T$ from its comparison matrix. The individual priority vectors are then synthesized into the group priority vector $\mathbf{w} = (w_1,\ldots,w_n)^T$ using the weighted arithmetic mean (WAM) (2) or weighted geometric mean (WGM–AIP) (3):

$$w_i = \sum_{k=1}^m \alpha_k w_i^k, \quad i = 1,\ldots,n; \tag{2}$$

$$w_i = \prod_{k=1}^m (w_i^k)^{\alpha_k}, \quad i = 1,\ldots,n \tag{3}$$

2.1. LW-AHP model

The Lehrer–Wagner (LW) model (Lehrer & Wagner, 1981) was adopted for the AHP by Regan et al. (2006). In this study, it was assigned as the LW-AHP model. The base of this model was placed in the philosophy of negotiation (Regan et al., 2006) and used for the AIP. The initial priority vectors ${}^0\mathbf{w}^k = ({}^0w_1^k,\ldots,{}^0w_n^k)^T$ $k = 1,\ldots,m$ were derived by the eigenvector method from DMs' comparison matrices. They were revised according to weights of respect, w_s^{ij}, which were based on the strength of the differences between the priorities of DMs for each criterion (or alternative) s, $s = 1,\ldots,n$.

$$w_s^{ij} = \frac{1 - |{}^0w_s^i - {}^0w_s^j|}{\sum_{j=1}^m (1 - |{}^0w_s^i - {}^0w_s^j|)}, \tag{4}$$

The weights are gathered in the matrices of weights of respect $W_s = (w_s^{ij})_{m\times m}$. Let ${}^0P_s$ denote the vector of DMs' priorities of the criterion s: ${}^0P_s = ({}^0w_s^1,\ldots,{}^0w_s^m)^T$. The updated priorities of the criterion s after the first round of the aggregation result in ${}^1P_s = W_s{}^0P_s = ({}^1w_s^1,\ldots,{}^1w_s^m)^T$. The process of aggregation was repeated with the same weights of respect: ${}^rP_s = (W_s)^r\,{}^0P_s$. As r approaches infinity, the revised priorities of criterion s converged towards the final priority ${}^cw_s = {}^cw_s^1 = \ldots = {}^cw_s^m$, which was equal for all DMs and where c was the number of iterations needed to reach the convergence.

2.2. GWLS model

Sun and Greenberg (2006) proposed a GWLS model for deriving group priorities:

$$\begin{aligned} &min \quad \sum_{k=1}^m \sum_{j=1}^n \sum_{i=1}^n \alpha_k (a_{ij}^k w_j - w_i)^2 \\ &subject\ to: \quad \sum_{i=1}^n w_i = 1, \\ &w_i > 0, \quad i = 1,\ldots,n \end{aligned} \tag{5}$$

and proved that the solution of model (5) is given by

$$w = C^{-1}\hat{\lambda}, \tag{6}$$

where

$$\begin{aligned} &C = \widetilde{A} + \widetilde{A}^T - \Lambda, \quad \widetilde{A} = (\tilde{a}_{ij})_{n\times n}, \quad \tilde{a}_{ij} = \sum_{k=1}^m \alpha_k a_{ij}^{(k)}, \\ &\hat{a}_{ij} = \sum_{k=1}^m \alpha_k \left(\left(a_{ij}^{(k)}\right)^2 + 1 \right), \quad \eta_j = \sum_{i=1}^n \hat{a}_{ij}, \quad \Lambda = diag(\eta_1, \eta_2, \ldots, \eta_n) \\ &\text{and} \quad C^{-1} = (\bar{c}_{ij})_{n\times n}, \quad \hat{\lambda} = \left(\frac{\lambda}{2}, \frac{\lambda}{2}, \ldots, \frac{\lambda}{2}\right)^T, \quad \lambda = 2/\left(\sum_{i=1}^n \sum_{j=1}^n \bar{c}_{ij}\right). \end{aligned} \tag{7}$$

2.3. PD&R model

Huang et al. (2009) proposed a group AHP model considering the differences of preference among criteria (or alternatives) and the ranks of the criteria (or alternatives) for each DM. The priority vector of DM k, $k = 1,\ldots,m$ was originally derived by the logarithmic least squares method (Crawford & Williams, 1985), but we used