Edge effects of BEMD improved by expansion of support-vector-regression extrapolation and mirror-image signals

Feng-Ping An[a], Da-Chao Lin[b],[*], Ying-Ang Li[c], Xian-Wei Zhou[a]

[a] School of Computer and Communication Engineering, University of Science and Technology Beijing, Beijing 100083, China
[b] Department of Civil Engineering, North China Institute of Science and Technology, Yanjiao, Beijing 101601, China
[c] The Graduate School, North China Institute of Science and Technology, Yanjiao, Beijing 101601, China

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ABSTRACT

In the operation of bidimensional empirical mode decomposition, expansion with mirror-image signals is an effective approach to weaken the edge effect. To meet the basic requirement that mirrors should be placed at the extrema, however, there is a problem to make full use of the information involved in the original signal. To address this problem, we propose an approach with the expansion of both support-vector-regression (SVR) extrapolation and mirror-image signals, in which the extrema are captured from the data of SVR extrapolation. The SVR model is constructed with the support vector method (SVM) based on the original signal data. Its extrapolation results in the estimation of the signal data beyond the edge for capturing the extrema so that the information of the original signal can be fully used in locating the mirror. Once all of these extrema points are determined, the traditional mirror expansion method is used and finally edge effects of the BEMD are eliminated. Results from numerical experiments show that the proposed approach has a good capability of improving edge effects of the BEMD operation process, and the reconstruction image from the decomposed components of the intrinsic mode function (IMF) confirms its high coherency with the original one.

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1. Introduction

The empirical mode decomposition (EMD) is a non-stationary time-frequency analysis technique proposed by Huang et al. in 1998 [1]. Compared with the other time-frequency signal processing methods such as short-time Fourier transform and wavelet transform, this method does not rely on any *priori* basic functions and leads to an adaptive decomposition process by its own characteristics of the data, which should be better at revealing signal features involved in the time-frequency localization behavior. As an adaptive signal analysis technique different from traditional ones, the EMD has been successfully applied to various fields of earthquake [2], structural vibration [3], biotechnology [4], mechanical fault diagnosis [5,6] and atmosphere science [7], etc.

Nunes et al. [8] extended the EMD of one-dimensional case to the two-dimensional, and proposed an algorithm of bi-dimensional empirical mode decomposition (BEMD). Subsequently, their work was further modified [9], and began to put into use of the image signal processing. Up to this data, the modified BEMD have been

suggested to be used in the aspects of image compression [10,11], image de-noising [12], image scaling [13], image feature extraction [14] and others. However, the BEMD algorithm has not been so perfect yet and its applications are limited by some faults such as the treatment of edge effects [15], selection of the interpolation method [16], searching of the extrema points and determination of the stopping criteria [17]. Relatively, the influence of edge effects on decomposition results has a close relationship with the integrity of the algorithm itself, which may directly throw an obstacle to its development. Therefore, the edge effect of the BEMD should be a basic problem which needs careful consideration in the relevant research works.

Essentially, edge effects are characteristic of gradually propagating toward the center of the signal with the increase of decomposition levels, which may badly damage the reliability of the decomposed results. To solve this problem, many scholars have carried out some studies on one-dimensional signals in depth, and developed many treatment methods such as mirror expansion or extrema expansion [18–20], neural networks expansion [21], support vector machine prediction [22], waveform feature matching [1,23,24] and so on. Although certain success has obtained in solving the problem of edge effects, it is necessary to do a well-established theoretical analysis which would permit a roof of the

* Corresponding author. Tel.: +86 10 61594883; fax: +86 10 61594887.
  E-mail address: lindachao@ncist.edu.cn (D.-C. Lin).

condition of application for each of them and a concrete estimation how to judge the result of improving the edge effect for each of them. Furthermore, it has been found that the mirror expansion method should be one of the most effective method to mitigate the edge effect with a large number of practical examples and their analyses [25].

The mirror expansion method, that is, the expansion with mirror-image signals, is to add its mirror image to the signal to expand the signal. In this method, the mirror is usually placed at the extrema of the signal which is nearest to the edge. If it is not sure whether one point on the edge takes an extremum or not, a piece of the signal near it must be removed till the first extremum has been found. It should be the position that meets the requirement of placing the mirror at the extrema point. Obviously, in the case of too short decomposed signal or/and some important features involved in the removed data, the decomposition result will lose the reliability. Even so, removing a piece of the signal near the edge to the first extrema is still the best way to realize the expansion with a mirror-image signal. With this method, it is reasonable to suspect this treatment and a satisfactory practical result can be obtained for the signal with a short length or unsuitable interception.

Can we accurately predict some data beyond the edge by a certain approach? If we can, the extrema nearest the edge of the image may be found from them and used as the positions for placing the mirror so as to avoid the flaw that some pieces of the signal required are removed. Recently, an investigation on improving the end effect of decomposing a one dimensional signal through the EMD has show a clue to answer this question [26]. Considering the high generalization ability of the support vector method (SVM) [27,28], we can select small subset of the training data to solve multidimensional function estimation problems, whereas a large number of free parameters would be required in classical techniques. By using the data of a specific signal, the support-vector-regression (SVR) relation is constructed through the SVM, and then some data beyond the edge of the signal are estimated by the extrapolation of the SVR model. In this way, it leads to a concept of expanding the signal, called as the support-vector-regression extrapolation, by which less amount of data can be used to predict the trend of the data itself.

Here we propose a new approach which combines expansion of the SVR extrapolation with the mirror-image signal to treat the edge effect of the BEMD algorithm. The approach may be briefly summarized as follows: construct the SVR model of the original signal through the SVM; extrapolate the obtained SVR model toward the outside regions of the image to get the prediction data beyond the edge; determine the local extrema nearest to four sides by them; and finally expand the signal with respect to these extrema by using the mirror expansion method and map it into a circle signal free of edge. A numerical experiment shows that the proposed approach to improve the edge effect of the BEMD gives a better result than the others in suppressing edge effects during processing the actual image signals.

2. BEMD algorithm

After the implementation of BEMD was proposed by Nunes et al. [8], several versions of its algorithm have been developed for these years [29–34]. For an $m \times n$ bidimensional image $f(x,y)$, the BEMD algorithm in the process called 2D-sifting procedure may be summarized as follows:

(1) Initialize the image under consideration, to set $r_0(x,y) = f(x,y)$ as the input signal, and $j = 1$.
(2) Set $h_{k-1}(x,y) = r_{i-1}(x,y)$ and $k = 1$, and extract the extrema involved in the h_{k-1}.

(3) Interpolate between maxima and between minima, respectively, to get two envelope surfaces $e_{\max}(x,y)$ and $e_{\min}(x,y)$.
(4) Capture the mean envelope surface in terms of the upper and lower envelope surfaces, given by

$$m(x,y) = [e_{\max}(x,y) + e_{\min}(x,y)]/2 \tag{1}$$

(5) Extract the residual $h_k(x,y)$, which is equal to image $h_{k-1}(x,y)$ subtracting mean envelope $m(x,y)$, given by

$$h_k(x,y) = h_{k-1}(x,y) - m(x,y) \tag{2}$$

(6) Iterate on $h_k(x,y)$ until this latter can be considered as an IMF component according to the stopping criterion SD. Once this process is achieved the resulting signal is considered as a proper mode, denoted as $c_i(x,y) = h_k(x,y)$. If the stopping criterion SD is not satisfied, the iteration will continue by letting $k = k+1$ and returning to step (2). The threshold of SD is generally taken as 0.2–0.3, and the actual value of SD is determined by

$$\text{SD} = \sum_{x=0}^{m} \sum_{y=0}^{n} \frac{\left| h_{k-1}(x,y) - h_k(x,y) \right|^2}{h_{k-1}^2(x,y)} \tag{3}$$

(7) Calculate the residual $r_i(x,y)$ by the following relation

$$r_i(x,y) = r_{i-1}(x,y) - c_i(x,y) \tag{4}$$

(8) Take $r_i(x,y)$ as a new input signal and repeat steps (1)–(8) to obtain the next mode $c_{i+1}(x,y)$ if $r_i(x,y)$ has more than two extrema, or otherwise, terminate the decomposition process.

Finally, the original signal can be expressed as

$$f(x,y) = \sum_{j=1}^{J} c_j(x,y) + r_J(x,y) \tag{5}$$

For a one-dimensional signal, since the residual obtained by the EMD is usually a constant or a monotonic function reflecting the trend of the signal, one can neglect its influence on some analyses of the signal. However, for a two-dimensional image, the BEMD may result in the residual with high gray value. In principle, influence of the residual on the image itself cannot be ignored in practical applications.

3. Expansion with mirror-image signals in the BEMD

Since the image signal in most cases is short, serious edge effects could be found in the process of the BEMD operation. As the sifting process goes on continuously, this type of phenomena will become more and more clear, which may make the obtained IMFs badly distort. Therefore, in decomposing the image by the BEMD, it is necessary to have it a proper treatment. Such a treatment should be capable of efficiently mitigating or eliminating the edge effects associated with the decomposition process of the BEMD.

At the earlier time of the EMD development, Huang et al. proposed a method, called as characteristic wave matching method [4], to treat the end effects. After then, some new methods, such as mirror expansion [16–20], neural network prediction [21], SVM prediction [22] and gray system prediction [35], have been introduced in succession. However, these methods have not developed their 2D versions for images. Although a texture synthesis-based method was proposed [34], its result shows that the real 2D method may not be effective in most cases for