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Testing Greeks and price changes in the S&P 500 options and futures contract: A regression analysis

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ABSTRACT

We use a regression model to test observed price changes with Greeks as regressors. Greeks are computed using implied volatility, price-change implied volatility and historical volatility. We find sufficient evidence to reject model Greeks as unbiased responses to underlying price as well as sufficient evidence that the American version of binomial model results in biased estimates of price changes. We use options on the S&P 500 futures contracts and their underlying. We also evaluate the frequency of "wrong signs." Call prices and their underlying move in the opposite direction almost 10 percent of the time.

1. Introduction

In recent years, capital markets have experienced an increasing use of derivative instruments. The important factors allowing for this growth in derivative markets have been both the hedging requirements of investors as well as advances in the development of valuation models. The arbitrage-free pricing model was developed by Black and Scholes, 1973. Since then, new models that allow for additional variables, such as stochastic volatility (Hull and White, 1987; Heston, 1993; Stein and Stein, 1991), jumps (Merton, 1976; Bates, 1996) and discrete time GARCH models (Duan, 1995; Heston & Nandi, 2000) have been developed. The empirical research testing the ability of these models to correctly price derivatives is extensive. This includes the work of, for example, MacBeth and Merville (1979), Rubinstein (1985), Shastri and Tandon (1986), Whaley (1986), Bakshi, Cao, and Chen (1997), and Bakshi, Cao, and Chen (2000).

Most of the papers in the empirical literature test models using the absolute error or absolute percentage error in *prices* as the loss function. In this paper we present an empirical test of *price changes* instead of price levels. The advantage of this approach is its direct implications for delta hedging. The local price change of the option in response to a change in the underlying is captured by the first partial ("delta") of the option with respect to the underlying and a drift component that is due to convexity ("gamma") and time ("theta"). By allowing time change to be small but finite, we are able to separate the effects of gamma and theta.

We examine price changes in S&P 500 futures options using the American version of the binomial model (ABM). The performance of the price change model depends in large measure on how the Greeks are calculated. In addition to traditional volatility measures such as implied volatility and historical volatility, we compute Greeks using a volatility implied by price changes (price-change implied volatility). To be more specific, the price-change implied volatility is the volatility that equates the observed price changes to the model price changes (Hilliard and Li, 2010). This measure is analogous to implied volatility but the focus is on price changes instead of price levels. To avoid confusion, we use term price-change implied volatility for volatility implied by price changes and term price-level implied volatility for traditional implied volatility estimated from price levels.

The price change model is tested using parameters estimated out of sample. That is, we estimate parameters in the first period and then use these estimated parameters for the regressions in the second period. In addition to the binomial model, we also analyze price changes using Black's (1976) European pricing model for futures. We find that our regression results are largely unaffected regardless of whether we use Black's model or the ABM model. In any case, we report results using the ABM since options in our database are American.

The data used in this study are options on the S&P 500 futures and their underlying. Futures contracts are used because they are directly traded, highly liquid and do not suffer from staleness and more complex arbitrage considerations as does the basket of stocks underlying the S&P 500 spot contract. In addition, the futures contracts reflect the market's assessment of future dividends.

We find high R^2 in regressions of observed option price changes on changes explained by the ABM. One reason the R^2s are not even higher is partially due to the presence of "wrong" signs. Bakshi et al.

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(2000) in their empirical study of price changes in S&P 500 index options found that prices of call (put) options frequently move in the opposite (the same) direction as the prices of underlying. That is, the signs are "wrong." These apparent violations can be due to various reasons such as to stochastic volatility, omitted variables or the segmentation of the market for derivatives and their underlying. In addition, they may be caused by nonsynchronous trading or staleness of the data. Since our dataset is very extensive and S&P 500 futures options are traded very frequently, we examine whether similar violations can be found also in the S&P 500 futures contracts.

The rest of the paper is organized as follows. Section 2 develops the regression approach for the price change model and Section 3 provides a description of data. The regression results are summarized in Section 4. Section 5 compares sign violations in the S&P 500 futures options with the findings of Bakshi et al. (2000) in the S&P 500 options and Section 6 concludes.

2. The regression approach

Option prices are assumed to be driven by a single factor (the underlying) and the usual no-arbitrage assumptions. That is, the option price at time t (H_t) is a function of the underlying (F_t), the maturity (T), a constant interest rate (r) and volatility (σ_t):

$$H_t = H(F_t, T, r, \sigma_t). \tag{1}$$

This pricing equation is expanded by Ito's formula to give a regression setup where price changes are expressed as function of the underlying and time. For a single factor model, the Ito expansion for the local change in derivative price, say H, is written as

$$dH = H_F dF + H_t dt + \frac{1}{2} H_{FF} dF^2, \tag{2}$$

where $H_F \equiv \frac{\partial H}{\partial F}$, $H_{FF} \equiv \frac{\partial^2 H}{\partial F^2}$ and $H_t \equiv \frac{\partial H}{\partial t}$. If F follows a geometric Brownian motion with volatility σ, then $dF^2 = F^2\sigma^2 dt$.

In our regressions however, we work with small but non-local changes. Therefore we express the price change relation using the Taylor series expansion written as

$$\Delta H = H_F \Delta F + H_t \Delta t + \frac{1}{2} H_{FF} \Delta F^2 + O\left(\Delta^{\frac{3}{2}}\right), \tag{3}$$

where the change in time and state are denoted by Δt and ΔF, respectively. Because of possible confusion between Δ as the Greek gradient and Δ as a small change, the notation for a small change used hereafter is $d(\cdot)$ instead of $\Delta(\cdot)$, e.g., dF replaces ΔF.

Eq. (3) is the basis for our hypothesis regarding the Greeks. In fact, a similar approach using Ito expansion was originally used by Bakshi et al. (2000). They test the underlying delta and the stochastic volatility delta. Our approach differs with respect to the underlying in that we assume a finite Δt and thus retain the ΔF^2 term, allowing us to separate the effect of Γ (H_{FF}) and (H_t) θ in our regressions. We test two hypotheses. First, we posit that model Greeks are unbiased responses to changes in the underlying and time. Rejection of this hypothesis suggests that Greeks should be adjusted to improve hedging effectiveness. And second, a more global hypothesis is that the ABM model produces unbiased estimates of observed price changes. Both hypotheses are tested using a regression model.

With respect to the Greeks, our hypothesis is that $\Delta = H_F$, $\Gamma = H_{FF}$ and $\theta = H_t$. A testable version of Eq. (3) is written in regression form as

$$dH = \alpha + \beta_1 \Delta dF + \beta_2 \theta dt + \beta_3 \Gamma \frac{dF^2}{2} + \varepsilon, \tag{4}$$

where ε is the zero mean error term, $dH = H_1 - H_0$, $dF = F_1 - F_0$ and $dt \equiv t_1 - t_0$.[1] When regressed in this fashion, the test of the three separate nulls is that $\beta_1 = 1$, $\beta_2 = 1$ and $\beta_3 = 1$.

The null hypothesis that the model produces unbiased estimates of price change implies that

$$E[dH|dF, dt] = H_F dt + H_t dt + H_{FF} \frac{dF^2}{2}. \tag{5}$$

The regression model for this hypothesis is

$$dH = \alpha + \beta\left(\Delta dF + \theta dt + \Gamma \frac{dF^2}{2}\right) + \varepsilon \tag{6}$$

with null $\alpha = 0$ and $\beta = 1$.

Violations of assumptions that lead to rejection of the ABM (or Black) null have been addressed in the literature. These include stochastic process misspecification, parameter estimation errors, non-zero transaction costs, non-synchronous transactions, microstructure issues and hedging pressure errors. See, for example, Gârleanu, Pedersen, and Poteshman (2009) for a discussion of these violations. While we are aware of misspecification issues in Black's model, we do not choose to do a horserace between different models. We simply want to demonstrate the explanatory power of standard Greeks on option price changes. We investigate Black's model because of its simplicity, familiarity, and heavy use by practitioners. In fact, Berkowitz (2001) provides theoretical justification of the Practitioner Black–Scholes model as a reduced form approximation to a theoretically correct but unknown model. Christoffersen and Jacobs (2004) then empirically demonstrate that by aligning the estimation and evaluation functions, the Black–Scholes model can even outperform the Heston model.

To develop the test, option pricing models are parameterized at time t and all right hand side gradients Δ, θ, and Γ are computed. At time $t + dt$, the right hand side components dF and dF^2 are known so that the regression equation is completely specified. The dependent variable dH is the observed change in option price and the right hand side is the change explained by the option pricing model given changes in the state variables.

Absent statistical considerations, evaluating a price change model is the same as evaluating a price level model. In price level models, the underlying price is assumed known and model price is compared to the observed price. In price change models, the underlying price change is assumed known and model price change is compared to observed price change.

In the sections that follow, regressions are developed using the ABM model with Greeks computed directly from the binomial tree. Following Figlewski (2002), a naive model of option price change versus stock price change is also tested. Figlewski tests a *price level* version of the Black–Scholes model using regressions against an informationally passive benchmark. The benchmark used is the exercise value of the option, i.e., $Max\{0, F - K\}$, where F is the price of the underlying and K is strike price. As a minimum threshold, the model should best the informationally passive benchmark in R^2 or other measures of model validity. The same procedure can be used for evaluating models of *price change*. One informationally passive benchmark is a regression of option price change versus price change in the underlying. We expect that the ABM model will outperform this informationally passive benchmark (hereafter the "naive model").

3. Data

The data consists of the prices and contract specifications on futures options, their underlying and contemporaneous risk-free interest rates.

[1] Note that a multivariate form of the Ito expansion can be used to develop price change equations for options depending on multiple state variables.