



Optimal city hierarchy: A dynamic programming approach to central place theory [☆]

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Received 8 April 2009; final version received 11 March 2013; accepted 17 September 2014

Available online 30 September 2014

Abstract

Central place theory is a key building block of economic geography and an empirically plausible description of city systems. This paper provides a rationale for central place theory via a dynamic programming formulation of the social planner's problem of city hierarchy. We show that there must be one and only one immediate smaller city between two neighboring larger-sized cities in any optimal solution. If the fixed cost of setting up a city is a power function, then the immediate smaller city will be located in the middle, confirming the locational pattern suggested by Christaller [4]. We also show that the solution can be approximated by iterating the mapping defined by the dynamic programming problem. The main characterization results apply to a general hierarchical problem with recursive divisions.

[☆] We are grateful to an anonymous associate editor and three anonymous referees for their very thoughtful and helpful comments and suggestions that drastically improved this paper. For their helpful comments, we also thank Yi-Chun Chen, Tom Davidoff, Gilles Duranton, Yannis Ioannides, Samuel Kortum, Erzo G.J. Luttmer, Shin-Kun Peng, Will Strange, Shenghao Zhu, and the seminar participants at Academia Sinica, National Taiwan University, National University of Singapore, Singapore Management University, the 2007 Midwest Economic Theory Meetings, the 2008 Annual Meeting of the Urban Economics Association, the 2009 Econometric Society's North American Summer Meeting, the 2009 Annual Meeting of Society of Economic Dynamics, and the 2012 IRES Symposium. We also thank Paul Thompson for permission to use his central place theory graph. Frank Morgan acknowledges partial support from the National Science Foundation DMS 0803168. All errors are ours.

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<http://dx.doi.org/10.1016/j.jet.2014.09.018>

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JEL classification: R12; R13; C61

Keywords: Central place theory; City hierarchy; Dynamic programming; Principle of optimality; Fixed point

1. Introduction

Central place theory describes how a city hierarchy is formed from a featureless plain of farmers as consumers. It is a key building block of economic geography (King [17]) and dates back at least to Christaller [4]. Many have argued for its empirical plausibility as a description of city hierarchy (Fujita, Krugman, and Venables [11], Berliant [3], Mori, Nishikimi, and Smith [22], Mori and Smith [23]). Although original central place theory is not a rigorous economic theory based on incentives and equilibrium, many economists have found its insights appealing, and a few attempts have been made to formalize it, including those by Eaton and Lipsey [8], Quinzii and Thisse [25], Fujita, Krugman, and Mori [10], Tabuchi and Thisse [31], and Hsu [16].¹

The basic idea behind this theory is that goods differ in their degree of scale economies relative to market size. Goods for which this ratio is large, e.g., stock exchanges or symphony orchestras, will be found in only a few places, whereas goods for which the ratio is small, e.g., gas stations or convenience stores, will be found in many places. Moreover, large cities tend to have a wide range of goods, whereas small cities provide only goods with low scale economies. Naturally, small cities are in the market areas of large cities for those goods that they themselves do not provide. In Christaller's scheme, the *hierarchy property*² holds if larger cities provide all of the goods that smaller cities also provide and more.

In this paper, a *city system* is composed of multiple layers of cities, and cities of the same layer have the same functions, i.e., they host the same set of industries. The driving force behind the differentiation of cities is the heterogeneity of scale economies among goods, which is modeled by heterogeneity in the setup costs of production. In addition to the hierarchy property, another defining feature of city hierarchy in central place theory, that is called the *central place property*, is that there is only one next-layer city between neighboring larger cities and it is halfway in between. Christaller [4] calls this the *K = 3 market principle*.³ The city hierarchy described by central place theory (hereafter *central place hierarchy*) is a city system in which both the hierarchy and central place properties hold. Fig. 1 provides an illustration of such a city hierarchy in a one-dimensional geographic space.⁴

This paper takes aim at providing a rationale for central place theory via a social planner's problem. An innovative feature of this paper is that the social planner's problem is formulated

¹ Besides central place theory, another important theory of city hierarchy is Henderson's [13] type-of-cities theory, which emphasizes cities' roles in industrial specialization. Also see the extension to city growth by Rossi-Hansberg and Wright [26].

² This is often called the *hierarchy principle* in the literature.

³ On the plane, if there is always only one next-layer city located at the centroid of the equilateral triangle area in between three neighboring larger cities, then the ratio of the market areas is 3.

⁴ The vertical axis shows the range of goods produced and goods are indexed by some measure of the degree of scale economies, e.g., fixed cost of production, $y \in [0, \bar{y}]$, for some $\bar{y} > 0$. The hierarchy property implies that each city provides goods in $[0, y]$ for some y . Hence, a layer- i city provides goods in $[0, y_i]$, and obviously, $y_1 = \bar{y}$.

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