Expert Systems With Applications 42 (2015) 9318–9325

Contents lists available at ScienceDirect

Expert Systems With Applications

journal homepage: www.elsevier.com/locate/eswa

Evolving a linear programming technique for MAGDM problems with interval valued intuitionistic fuzzy information

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ARTICLE INFO

Keywords:
Multiattribute group decision making
Interval valued intuitionistic fuzzy sets
Variable transformation
Aggregation operators

ABSTRACT

It is believed that multi attribute decision making problem is an ill-defined and unstructured problem. This difficulty intensifies while considering the uncertainty of decision makers' information about the problem. In recent years, interval valued intuitionistic fuzzy sets (hereafter IVIFs) as a generalization of ordinal fuzzy sets, became a well-known and widely applied framework for dealing with uncertainty of decision making problems. However, the mathematical programming aspects of interval valued intuitionistic fuzzy sets besides their applications in decision making problems are neglected. To reinforce the mathematical programming approach in IVIF environment, an IVIF multi attribute group decision making problem is formulated as a linear programming model. Using a variable transformation and the notion of aggregation operators, the proposed model is transformed into an equivalent linear programming model solvable by common approaches. Application of the proposed method is represented in a group decision making problem and the results are compared with similar methods, proving the compatibility of the proposed method with previous ones. The solid and understandable logic with computational easiness are the main advantages of the proposed method. Solving interval valued intuitionistic fuzzy linear programming problems can be applied lucratively in other problems being formulated in this context.

1. Introduction

Multi criteria decision making is one of the most implicational fields of operations research. From a managerial perspective, decision making problems can be classified into two classes of planning and selection problems (Simon, 1977). Multi criteria decision making (henceforth MCDM) considers those problems where several criteria must be satisfied to make a desirable decision. MCDM is further divided to multi objective decision making (henceforward MODM) and multi attribute decision making (hereafter MADM) (Climaco, 1997). Usually, MODM considers the planning type problems, while MADM deals with selection problems.

An MADM problem can be defined formally as follow: Let $A = \{A_1, A_2, \ldots, A_m\}$ be a nonempty and finite set of decision alternatives and $C = \{C_1, C_2, \ldots, C_n\}$ is a finite set of goals, attributes or criteria, according to which desirabilityof an alternative is to be judged. The aim of MADM is to determine the optimal alternative with the highest degree of desirability respect to all relevant goals (Zimmerman, 1987).

Usually, MADM problems are characterized in the form of a decision matrix $D = [x_{ij}]$, where x_{ij} is the performance of alternative A_i regard to criterion C_j, and a weight vector $W = [w_1, w_2, \ldots, w_n]$, where w_j represents the importance of criterion C_j in decision making. In classic MADM problems, it is supposed that x_{ij} and w_j values are determined as crisp numbers. However, there are some reasons that human information is often incomplete and inexact. Yovits (1984) believed that uncertainty may occur due to partial or approximate information. In fact, most of our information about our surrounding phenomena is determined partially or approximately. Therefore, it seems necessary to follow some frameworks to cope with this uncertainty. Liu and Lin (2006) classified different frameworks of uncertainty in three categories of probability and statistics, grey system theory, and fuzzy set theory.

Fuzzy set theory (Zadeh, 1965) is one of the widely accepted frameworks regarding the uncertainty. Fuzzy sets are generalized

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http://dx.doi.org/10.1016/j.eswa.2015.07.067
0957-4174/

forms of classic sets in which a membership degree (function) is assigned to each element of a universal set, opposed with classic sets where a sharp distinction is assumed between membership and non-membership of elements. Fuzzy set theory is widely applied in decision making problems.

Grattan-Guinness (1976) and later Gau and Buehrer (1993) pointed that the presentation of a linguistic expression in the form of fuzzy set is not enough. Actually, it is so hard to determine an exact membership degree of elements and there is not any hesitation in determining. Considering the hesitancy, Atanassov (1986) introduced the notion of intuitionistic fuzzy set (IFS) as a generalization of the Zadeh's fuzzy sets. In addition to membership degree of each element in ordinal fuzzy sets, IFS assigns a degree of non-membership to each element. Subsequently, Atanassov and Gargov (1989) extended the interval valued intuitionistic fuzzy sets (IVIFS), where membership and non-membership degrees are defined as closed intervals.

One of the prominent applications of IVIFS is in the area of MADM or MAGDM problems. In such problems, the elements of matrix D and/or vector W are illustrated as IVIFNs. Some researchers developed well-known MADM methods in IVIF form. Tan (2011), Verma, Verma, and Mahanti (2001), Ye (2010), and Park, Park, Kwun, and Tan (2011) extended TOPSIS method under IVIF framework with different distance measures between IVIFNs and ideal and anti- ideal definitions. An IVIF extension of VIKOR method is presented by Park, Cho, and Kwun (2011). Razavi Hajiagha, Hashemi, and Zavadskas (2013) proposed an IVIF version of Complex Proportional Assessment (COPRAS) method. Zavadskas, Antucheviciene, Razavi Hajiagha, and Hashemi (2014) extended the weighted aggregated sum product assessment (WASPAS) method under IVIF environment. Zavadskas, Antucheviciene, Razavi Hajiagha, and Hashemi (2015) also extended the Multi-objective Optimization by Ratio Analysis plus Full Multiplicative Form (MULIMOORA) for MAGDM with IVIF information. Chen (2015) proposed an IVIF version of preference ranking organization method for enrichment evaluations (IVIF-PROMETHEE) and elaborated on its applications for MAGDM.

Beyond the extension of previous models, some recent developed methods are proposed for IVIF MADM problems. Li (2010a) developed a nonlinear programming methodology based on TOPSIS to solve MADM problems using ratings of alternatives on attributes and weights of attributes expressed with IVIFSs. Li (2010b) used the concept of relative closeness coefficients and constructed a pair of nonlinear fractional programming models being transformed into two simpler auxiliary linear programming models in order to calculate the relative closeness coefficient of alternatives to the IVIF positive ideal solution, being employed to generate ranking order of alternatives. Li (2011) proposed a closeness coefficient based nonlinear programming method for solving IVIF MADM problems. Lakshmana Gomathi Nayagam, Muralikrishnan, and Sivaraman (2011) introduced a new method of IVIFNs ranking and a method of MADM upon IVIF information. Yue (2011) developed a new approach for measuring decision makers' weights in IVIF group decision making problems on the concept of ideal decision of a group and the similarity measure between each individual decision and ideal decision. Chen, Lee, Liu, and Yang (2012) solved MADM problems on the basis of interval-valued intuitionistic fuzzy weighted average operator and a new defined fuzzy ranking method for intuitionistic fuzzy values. Chen, Yang, Yang, Sheu, and Liau (2012) presented a method for multi criteria fuzzy decision making under IVIFS, where a new method was proposed for ranking IVIF values; afterwards, the results were used for multi criteria decision making problems. Wang and Li (2012) introduced group consistency and inconsistency indices and determined unified attribute weights and an interval-valued intuitionistic fuzzy positive ideal solution using an auxiliary linear programming model. The obtained weights were then used to calculate distances of alternatives from positive ideal solution. Ye (2013) used Entropy weight models to determine the weights of both experts and attributes from IVIF decision matrices, following that the evaluation formulas of weighted correlation coefficients between alternatives and the ideal alternative was denoted. The alternatives were ranked regard to the values of the weighted correlation coefficients for IFSs or IVIFSs. Wang and Liu (2013) introduced some Einstein geometric operators on IVIFS and applied an IVIF Einstein hybrid weighted geometric based approach to solve MADM problems. Wan and Dong (2014) used the notion of 2-dimensional random vector to rank IVIFNs. Following Karnik–Mendel algorithm, they defined ordered weighted average operator and hybrid weighted average operator for IVIFNs and applied these operators for MAGDM. Jin, Pei, Chen, and Zhou (2014) proposed the IVIF continuous weighted entropy and developed an approach for MAGDM problems conditional upon the weighted relative closeness and IVIF attitudinal expected score function. Wan, Xu, Wang, and Dong (2015) developed a method of solving MAGDM problems with IVIF information. In their method, they determined the weight of each decision maker with respect to every attribute; subsequently, the collective decision matrix is transformed into an interval matrix using the risk coefficient of DMs. A multi-objective interval-programming model is then solved to derive the attribute weights. Ultimately, the comprehensive interval values of alternatives are used to rank the alternatives.

The aim of this paper is to propose a formulation of multi attribute group decision making (MAGDM) problems. In this paper, the MAGDM problem is formulated in the form of a linear programming problem and then, a novel approach is proposed to solve this problem. Applicability of the proposed method is presented in a numerical example and the results are compared with some previously presented methods. This paper is organized as follows. A brief overview of interval valued intuitionistic fuzzy sets and required concepts are given in Section 2. The considered problem and its formulation are expressed in Section 3. Section 4 explains the proposed approach for solving the problem. In Section 5, application of the proposed approach in multi attribute group decision making is explained and the results of proposed method are compared with some previous extended methods. Finally, Section 6 makes some conclusions.

2. Interval valued intuitionistic fuzzy sets

Atanassov and Gargov (1989) generalized the IFS concept to interval valued intuitionistic fuzzy sets (IVIFSs). Let $D[0,\ 1]$ be the set of all closed subintervals of the interval [0, 1]. Let X be a given non-empty set. An IVIFS in X is an expression given by $\tilde{A} = \{x, \mu_{\tilde{A}}(x), \upsilon_{\tilde{A}}(x) | x \in X\}$, where $\mu_{\tilde{A}} : X \to D[0,\ 1]$, $\upsilon_{\tilde{A}} : X \to D[0,\ 1]$ under the condition $0 < sup_x \mu_{\tilde{A}}(x) + sup_x \upsilon_{\tilde{A}}(x) \le 1$.

The intervals $\mu_{\tilde{A}}(x)$ and $\upsilon_{\tilde{A}}(x)$ denote the membership and non-membership degrees of the element x to the set A. Thus, for each $x \in X$, $\mu_{\tilde{A}}(x)$ and $\upsilon_{\tilde{A}}(x)$ are closed intervals whose lower and upper end points are denoted by $\mu_{AL}(x)$, $\mu_{AU}(x)$, $\upsilon_{AL}(x)$, and $\upsilon_{AU}(x)$.

The IVIFS A is denoted by

$$A = \{ \langle x, [\mu_{AL}(x), \mu_{AU}(x)], [\upsilon_{AL}(x), \upsilon_{AU}(x)] \rangle | x \in X\} \tag{1}$$

where $0 < \mu_{AU}(x) + \upsilon_{AU}(x) \le 1$, $\mu_{AL}(x),\ \upsilon_{AL}(x) \ge 0$. For convenience, an IVIFS value is denoted by $\tilde{A} = ([a, b], [c, d])$ and called as an interval valued intuitionistic fuzzy number (IVIFN).

If $\tilde{A}_1 = ([a_1, b_1], [c_1, d_1])$ and $\tilde{A}_2 = ([a_2, b_2], [c_2, d_2])$ be any two IVIFNs, their operational laws are defined as follows (Xu, 2007):

$$\tilde{A}_1 + \tilde{A}_2 = ([a_1 + a_2 - a_1 a_2, b_1 + b_2 - b_1 b_2], [c_1 c_2, d_1 d_2]) \tag{2}$$

$$\tilde{A}_1 \cdot \tilde{A}_2 = ([a_1 a_2, b_1 b_2], [c_1 + c_2 - c_1 c_2, d_1 + d_2 - d_1 d_2]) \tag{3}$$

$$\lambda \tilde{A}_1 = \left(\left[1 - (1 - a_1)^{\lambda}, 1 - (1 - b_1)^{\lambda}\right], \left[c_1^{\lambda}, d_1^{\lambda}\right]\right),\ \lambda \ge 0 \tag{4}$$