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# Comparison of nine meta-heuristic algorithms for optimal design of truss structures with frequency constraints

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ABSTRACT

Structural optimization with frequency constraints is a challenging class of optimization problems characterized by highly non-linear and non-convex search spaces. When using a meta-heuristic algorithm to solve a problem of this kind, exploration/exploitation balance is a key feature to control the performance of the algorithm. An excessively exploitative algorithm might focus on certain areas of the search space ignoring the others. On the other hand, an algorithm that is too explorative overlooks high quality solutions as a result of not performing adequate local search.

This paper compares nine multi-agent meta-heuristic algorithms for sizing and layout optimization of truss structures with frequency constraints. The variation of the diversity index during the optimization history is analyzed in order to inspect exploration/exploitation properties of each algorithm. It appears that there is a significant relationship between the algorithm efficiency and the evolution of the diversity index.



## 1. Introduction

In low frequency vibration problems, the response of the structure depends in most part on fundamental frequencies and mode shapes [1]. Therefore, the dynamic behavior of a structure can be controlled by constraining these parameters. Minimizing the mass of a structure for which some natural frequencies should be upper and/or lower-bounded corresponds to formulate a structural optimization problem with frequency constraints.

Structural optimization with frequency constraints was first introduced by Bellagamba and Yang [2] in 1980s, and since then has received considerable attention by optimization experts that developed several algorithms. Optimization problems including both sizing and layout variables are very demanding especially when frequency constraints are lower-bounded [3]. Solution of frequency-constrained problems including both sizing and layout variables is still an open issue.

Sergeyev and Mroz [4] pointed out that natural frequencies of a structure are much more sensitive to layout modifications than to the size modifications. This may be because layout modifications often imply mode switches which in turn result in significant changes in natural frequencies. In addition, sizing and layout variables usually are of different nature and have different orders of magnitude. These facts make the combined sizing-layout optimization of structures with frequency constraints a challenging problem of its kind, including several local optima. Since frequency constraints are highly non-linear, non-convex and implicit functions with respect to design variables, this class of problems is very indicative to evaluate the performance of meta-heuristic algorithms. It is very important to design an optimization engine with a proper balance between diversification (i.e. global exploration of the search space) and intensification (i.e. local exploitation of the best solutions found) [5].

A short survey on the literature of optimization with frequency constraints is now provided. Konzelman [6] utilized dual methods and approximation concepts. Grandhi and Venkayya [7] used an optimality criterion based on the uniform Lagrangian density for resizing and scaling to locate boundary constraints. Sedaghati et al. [8] used an integrated finite element force method for frequency determination and mathematical programming to optimize truss and frame structures. Wang et al. [9] formed an optimality criterion via differentiation of the Lagrangian function: 3D truss structures were optimized including simultaneously sizing and layout variables; the initial design was chosen so to have the minimum weight increment. Lingyun et al. [10] hybridized the simplex search method and genetic algorithms to formulate the niche genetic hybrid algorithm (NGHA) that was applied to truss sizing/layout optimization problems. Lin et al. [11] minimized the weight of structures subject to static and dynamic constraints with a bi-factor algorithm based on Kuhn-Tucker conditions.

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As far as it concerns meta-heuristic algorithms, Gomes [3] used Particle Swarm Optimization (PSO) to solve sizing/layout optimization of trusses under frequency constraints. Kaveh and Zolghadr utilized the Charged System Search (CSS) method and its enhanced form [12], and a hybridized CSS–BB-BC with trap recognition capability [13]. Furthermore, they developed the Democratic Particle Swarm Optimization (DPSO) [14] and the hybridized PSRO [15] algorithms to improve exploration capability of PSO. Improving exploration ability of PSO has been an active research topic in recent years [16–18].

This paper will compare the performance of nine meta-heuristic algorithms for weight minimization of truss structures subject to frequency constraints: Particle Swarm Optimization (PSO) introduced by Kennedy and Eberhart [19], Harmony Search (HS) developed by Geem et al. [20], Big Bang–Big Crunch (BB–BC) presented by Erol and Eksin [21], Firefly Algorithm (FA) introduced by Yang [22,23], Charged System Search (CSS) developed by Kaveh and Talatahari [24], Cuckoo Search (CS) formulated by Yang and Deb [25], Enhanced Ray Optimization (ERO) a slightly improved version of Ray Optimization (RO) introduced by Kaveh and Khayatazad [26], Democratic Particle Swarm Optimization (DPSO) [14], and hybridized Particle Swarm Ray Optimization (PSRO) [15] both proposed by Kaveh and Zolghadr. The performance of each algorithm is evaluated on the basis of the convergence behavior observed in the optimization process. For that purpose, a diversity index is defined. Exploration/exploitation behavior is related with algorithm performance by analyzing the trend of the diversity index observed in the optimization process.

The article is structured as follows. Section 2 presents the statement of the minimum weight problem for a truss structure subject to frequency constraints. The optimization algorithms compared in this study are concisely reviewed in Section 3. In Section 4, the relative merits of the algorithms are assessed by solving five optimization problems. Some concluding remarks are provided in Section 5.

## 2. Formulation of the optimization problem

The mixed sizing-layout optimization problem for a truss structure subject to frequency constraints where the objective is to minimize the weight of the structure can be stated as follows:

$$\begin{aligned}
&\text{Find } X = [x_1, x_2, x_3, \ldots, x_n] \\
&\text{to minimize } P(X) = f(X) \times f_{penalty}(X) \\
&\text{subject to} \\
&\omega_j \leqslant \omega_j^* \text{ for some natural frequencies } j \\
&\omega_k \geqslant \omega_k^* \text{ for some natural frequencies } k \\
&x_{imin} \leqslant x_i \leqslant x_{imax}
\end{aligned} \tag{1}$$

where $X$ is the vector of the design variables, including both nodal coordinates and cross-sectional areas; $n$ is the number of optimization variables which depends on element grouping; $f(X)$ is the cost function, which is taken as the weight of the structure in a weight optimization problem; $f_{penalty}(X)$ is the penalty function, which is used to make the problem unconstrained. When some constraints corresponding to the response of the structure are violated in a particular solution, the penalty function magnifies the weight of the solution by taking values bigger than one; $P(X)$ is the penalized cost function or the objective function to be minimized. $\omega_j$ is the $j$th natural frequency of the structure with the corresponding upper bound $\omega_j^*$, while $\omega_k$ is the $k$th natural frequency of the structure with the corresponding lower bound $\omega_k^*$. $x_{imin}$ and $x_{imax}$ are the lower and upper bounds for the design variable $x_i$, respectively.

The cost function can be expressed as:

$$f(X) = \sum_{i=1}^{nm} \rho_i L_i A_i \tag{2}$$

where $nm$ is the number of structural members; $\rho_i$, $L_i$, and $A_i$ are the material density, length, and cross-sectional area of $i$th element.

The penalty function is defined as:

$$f_{penalty}(X) = (1 + \varepsilon_1 . \upsilon)^{\varepsilon_2}, \quad \upsilon = \sum_{i=1}^{q} \upsilon_i \tag{3}$$

where $q$ is the number of frequency constraints. The values for $\upsilon_i$ can be considered as:

$$\upsilon_i = \begin{cases} 0 & \text{if the ith constraint is satisfied} \\ \left|1 - \frac{\omega_i}{\omega_i^*}\right| & \text{else} \end{cases} \tag{4}$$

The parameters $\varepsilon_1$ and $\varepsilon_2$ determine the degree to which a violated solution should be penalized. In this study $\varepsilon_1$ is taken as unity, and $\varepsilon_2$ starts from 1.5 and then linearly increases to 6 for all the considered test problems. Such a scheme penalizes the unfeasible solutions more severely as the optimization process proceeds. As a result, in the early stages the agents are free to explore the search space, but at the end they tend to choose solutions without violation.

## 3. Optimization algorithms

All the algorithms considered here are multi-agent meta-heuristic methods. These algorithms start with a set of randomly selected candidate solutions of the optimization problem and attempt to improve the quality of the set based on a cost function. According to a series of simple rules, mainly inspired from nature, the existing solutions are perturbed iteratively in order to improve their cost function values. In this section the main rules of the algorithms are briefly summarized.

### 3.1. Particle swarm optimization (PSO)

Particle Swarm Optimization (PSO) is a meta-heuristic algorithm which mimics the social behavior of certain species of animals like birds flocking and fishes schooling. This algorithm, for which many modified variants have been proposed, was originally introduced by Kennedy and Eberhart [19]. In canonical PSO the next position of a particle is determined using the particle's best experience (local best solution) and the whole swarm's best experience (global best solution). Based on these two sources of information, the velocity vector for the $i$th particle in iteration $k+1$ can be formulated as follows:

$$\upsilon_{i,j}^{k+1} = \chi[\omega \upsilon_{i,j}^k + c_1 r_1 (xbest_{i,j}^k - x_{i,j}^k) + c_2 r_2 (xgbest_j^k - x_{i,j}^k)] \tag{5}$$

where, $\upsilon_{i,j}^k$ is the velocity or the amount of change of the design variable $j$ for the $i$th particle in $k$th iteration; $x_{i,j}^k$ is the current value of the $j$th design variable of the $i$th particle, $xbest_{i,j}^k$ is the best value of the design variable $j$ ever found by the $i$th particle; $xgbest_j^k$ is the best value of the design variable $j$ experienced by the entire swarm up to $k$th iteration; $r_1$ and $r_2$ are two random numbers uniformly distributed in the range (0, 1); $c_1$ and $c_2$ are two parameters representing the particle's confidence in itself and in the swarm, respectively; these parameters, which determine the particle's inclination toward local and global best solutions, are suggested to be taken as 2 by Kennedy and Eberhart [19]; $\boldsymbol{\omega}$ is the inertia weight for the previous iteration's velocity and controls the exploration tendency of the algorithm. In reference [3] which uses this algorithm for the same optimization problems this parameter is defined as:

$$\boldsymbol{\omega} = 0.4[1 + min(cov, 0.6)] \tag{6}$$

where $cov$ is the coefficient of variation of the objective functions of swarm particles. The constriction factor parameter $\chi$, which is used