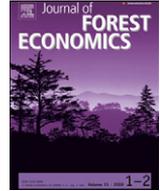




Contents lists available at ScienceDirect

Journal of Forest Economics

journal homepage: [www.elsevier.de/jfe](http://www.elsevier.de/jfe)



## Contrasting two approaches in real options valuation: Contingent claims versus dynamic programming

M.C. Insley<sup>a,\*</sup>, T.S. Wirjanto<sup>b</sup>

<sup>a</sup> Department of Economics, University of Waterloo, Waterloo, Ontario, Canada N2L 3G1

<sup>b</sup> School of Accounting and Finance and the Department of Statistics and Actuarial Science, University of Waterloo, Waterloo, Ontario, Canada N2L 3G1

### ARTICLE INFO

#### Article history:

Received 28 July 2008

Accepted 16 November 2009

#### JEL classification:

Q23

D81

G11

#### Keywords:

Optimal tree harvesting

Real options

Contingent claims

Dynamic programming

### ABSTRACT

This paper compares two well-known approaches for valuing a risky investment using real options theory: contingent claims (CC) with risk neutral valuation and dynamic programming (DP) using a constant risk adjusted discount rate. Both approaches have been used in valuing forest assets. A proof is presented which shows that, except under certain restrictive assumptions, DP using a constant discount rate and CC will not yield the same answers for investment value. A few special cases are considered for which CC and DP with a constant discount rate are consistent with each other. An optimal tree harvesting example is presented to illustrate that the values obtained using the two approaches can differ when we depart from these special cases to a more realistic scenario. We conclude that for real options problems the CC approach is preferred when data exists (such as futures prices) that allow the estimation of the market price of risk or convenience yield. Even when such data do not exist we argue that the CC approach is preferred as it has the advantage of allowing the individual specification of the prices of different sources of risk.

© 2009 Elsevier GmbH. All rights reserved.

### Introduction

Over the past two decades, developments in the theory and methodology of financial economics have been applied to advantage to general problems of investment under uncertainty. The well known book by Dixit and Pindyck (1994) draws the analogy between valuing financial options and

\* Corresponding author. Tel.: +1 519 888 4567.

E-mail addresses: [minsley@uwaterloo.ca](mailto:minsley@uwaterloo.ca) (M.C. Insley), [twirjant@uwaterloo.ca](mailto:twirjant@uwaterloo.ca) (T.S. Wirjanto).

investments in real assets or real options which involve irreversible expenditures and uncertain future payoffs depending on one or more stochastic underlying variables. Natural resource investments, including forestry, provide a good application of real options theory as their value depends on volatile commodity prices and they entail decisions about the timing of large irreversible expenditures.<sup>1</sup>

Two particular approaches used in the real options literature are dynamic programming (DP) and contingent claims (CC). DP is an older approach developed by Bellman and others in the 1950s and used extensively in management science. DP involves formulating the investment problem in terms of a Hamilton–Jacobi–Bellman (HJB) equation and solving for the value of the asset by backward induction using a discount rate which reflects the opportunity cost of capital for investments of similar risk. In practice dynamic programming typically involves adopting an exogenous constant discount rate.

The contingent claims approach has its origins in the seminal papers of Black and Scholes (1973) and (Merton, 1971, 1973) and is now standard in many finance texts.<sup>2</sup> This approach assumes the existence of a sufficiently rich set of markets in risky assets so that the stochastic component of the risky project under consideration can be exactly replicated. Through appropriate long and short positions, a riskless portfolio can be constructed consisting of the risky project and investment assets which track the project's uncertainty. In equilibrium with no arbitrage opportunities, this portfolio must earn the risk free rate of interest, which allows the value of the risky project to be determined. The no-arbitrage assumption avoids the necessity of determining the appropriate risk adjusted discount rate. However, if a portion of the return from holding the risky asset is due to an unobservable convenience yield, it is still necessary to estimate either that convenience yield or a market price of risk, which is often problematic.<sup>3</sup>

Both CC and DP have been used in the natural resources literature. For example Slade (2001) and Harchaoui and Lasserre (2001) use a contingent claims approach to value mining investments. In the forestry economics literature, the DP approach has generally dominated. An exception is Morck et al. (1989) who use a CC approach along with an assumed convenience yield for an application in forestry. In those forestry papers that use a DP approach, there is rarely much discussion of the choice of discount rate. Sometimes a risk neutral setting is explicitly assumed allowing use of a riskfree discount rate; other times a rate is adopted without explanation. A selection of papers that use the dynamic programming approach include Clarke and Reed (1989), Haight and Holmes (1991), Thomson (1992), Yin and Newman (1997), Plantinga (1998), Gong (1999), Insley (2002), and Insley and Rollins (2005). Alvarez and Koskela (2006, 2007) deal with risk aversion by explicitly modelling the decision maker's subjective utility function.

One reason for the dominance of the DP approach in the forestry literature is likely due to the difficulty in estimating the convenience yield. In theory futures prices could be used to obtain such an estimate. Futures markets do exist for lumber, however, currently the maturity of futures contracts is less than one year, while the typical optimal harvesting problem is applied to a very long lived investment, with stands of trees maturing over 40 to 70 years.<sup>4</sup>

---

<sup>1</sup> Examples of applications of real options theory to natural resources include Paddock et al. (1988), Brennan and Schwartz (1985), Schwartz (1997), Slade (2001) and references therein, Harchaoui and Lasserre (2001), Mackie-Mason (1990), Saphores (2000), and papers contained in Schwartz and Trigeorgis (2001). A review of the empirical significance of real options in valuing mineral assets is contained in Davis (1996).

<sup>2</sup> See Hull (2006) and Ingersoll (1987) for example. Dixit and Pindyck (1994) and Trigeorgis (1996) present contingent claims in a real options context.

<sup>3</sup> Dixit and Pindyck (1994) discusses the convenience yield in detail. It represents a return that accrues to the holder of the physical asset but not the holder of an option on the asset. For commodities such as copper, oil, or lumber the convenience yield represents the benefits of holding inventory rather than having to purchase the commodity in the spot market.

<sup>4</sup> There have been papers addressing this issue for other commodities including Gibson and Schwartz (1990) and Schwartz and Smith (2000). Another difficulty with estimating a convenience yield from a timber investment is that a stand of trees produces several different products such as lumber and paper whereas futures are traded in lumber only.

متن کامل مقاله

دریافت فوری ←

**ISI**Articles

مرجع مقالات تخصصی ایران

- ✓ امکان دانلود نسخه تمام متن مقالات انگلیسی
- ✓ امکان دانلود نسخه ترجمه شده مقالات
- ✓ پذیرش سفارش ترجمه تخصصی
- ✓ امکان جستجو در آرشیو جامعی از صدها موضوع و هزاران مقاله
- ✓ امکان دانلود رایگان ۲ صفحه اول هر مقاله
- ✓ امکان پرداخت اینترنتی با کلیه کارت های عضو شتاب
- ✓ دانلود فوری مقاله پس از پرداخت آنلاین
- ✓ پشتیبانی کامل خرید با بهره مندی از سیستم هوشمند رهگیری سفارشات