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Robust tracking error portfolio selection with worst-case downside risk measures

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ABSTRACT

This paper proposes downside risk measure models in portfolio selection that captures uncertainties both in distribution and in parameters. The worst-case distribution with given information on the mean value and the covariance matrix is used, together with ellipsoidal and polytopic uncertainty sets, to build-up this type of downside risk model. As an application of the models, the tracking error portfolio selection problem is considered. By lifting the vector variables to positive semidefinite matrix variables, we obtain semidefinite programming formulations of the robust tracking portfolio models. Numerical results are presented in tracking SSE50 of the Shanghai Stock Exchange. Compared with the tracking error variance portfolio model and the equally weighted strategy, the proposed models are more stable, have better accumulated wealth and have much better Sharpe ratio in the investment period for the majority of observed instances.

1. Introduction

In his seminal work, Roll (1992) proposed an active portfolio selection model in which the variance of tracking error of portfolio is minimized and is used to measure how closely a portfolio follows the index. Motivated by Roll's work, many researchers have worked on the active portfolio selection problems (see Alexander and Baptista, 2008; Jorion, 2003; Rudolf et al., 1999; Zhao, 2007) and for more recent studies, refer to Chen et al. (2011), Chiu and Wong (2011), Guedj and Huang (2009), Glode (2011) and Liu and Xu (2010). However, improvement is limited when the variance is used as a risk measure since the gain and loss are symmetric to the mean value in the variance. Markowitz (1959) was aware of the shortcoming of using variance and proposed using the semivariance of a portfolio to control risk. Bawa (1975), Bawa and Lindenberg (1977) and Fishburn (1977) introduced a class of downside risk measure known as the lower partial moment (LPM) to better suit different risk profits of the investors. Since the LPM can be used to control the loss of portfolio, it has become a popular research topic in theory and practice (see Chen et al., 2011; Grootveld and Hallerbach, 1999; Harlow, 1991; Yu et al., 2006).

Let ξ be a random variable and ρ be constant. The LPM of random variable ξ with respect to ρ can be expressed as

$$\mathrm{LPM}_m(\rho) = \mathbb{E}[((\rho-\xi)_+)^m]$$

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where $m \geq 0$ is a constant that can reflect the attitude of investor to risk and $(\cdot)_+ = \max(\cdot, 0)$. Particularly, for $m = 0,1$, $\text{LPM}_m(\rho)$ can be computed by

$$\text{LPM}_m(\rho) = \mathbb{E}[((\rho-\xi)_+)^m] = \begin{cases} \mathbb{P}\{\xi \leq \rho\}, & m=0; \\ \mathbb{E}[(\rho-\xi)_+], & m=1, \end{cases} \tag{1}$$

where $\mathbb{E}[\cdot]$ and $\mathbb{P}\{\cdot\}$ express the expectation and probability of a random variable, respectively. Thus, LPM_0 is no more than the probability of random variable ξ falling below the target ρ and LPM_1 is the expected shortfall of ξ falling below ρ.

There are two difficulties an investor would face if LPM were used to control the loss of a portfolio. One is that the exact expression of LPM is almost impossible without knowing the exact distribution of a random variable ξ. Another is that the estimating errors of the mean value and the variance of ξ cannot be avoided if the sample estimate of ξ is used in practice. To overcome these difficulties, in the last decade portfolio models based on the robust optimization technique have become a focal point of many researchers (see a recent survey Fabozzi et al., 2010 for details). A good case in point is Ben-Tal and Nemirovski (1998). Two cases are commonly considered in this regard. One is parameter uncertainty, e.g., see, Costa and Paiva (2002), Erdoğgany et al. (2004), Glabadanidis (2010), Goldfarb and Iyengar (2003), Ling and Xu (2012), Lu (2011) and Zymler et al. (2011) and another is distribution uncertainty, e.g., see, Huang et al. (2008); Liu (2011); Miao and Wang (2011) and Zymler et al. (2013). The two types of models, however, share a common weak point: the returns of risk assets in these models are assumed to either follow a known distribution with uncertain parameters, or have the exact estimations of parameters with uncertain distributions. In other words, they do not cover the case where both uncertainties arise together.

Our research in this paper will deal simultaneously with the uncertainties from both distribution and parameters. To capture the distribution uncertainty, we consider a worst-case downside risk model, and to capture the parameter uncertainty, we add more flexibilities to the model by allowing the parameters to change in either an ellipsoidal or a polyhedral uncertainty set. The proposed model is different from that of Zhu and Fukushima (2009) and Zhu et al. (2009), in which they proposed a robust framework based on downside risk constraints with discrete distribution. In order to enhance computability, we consider an equivalent semidefinite programming (SDP) formulation by using convex duality principle. By relaxing some vector decision variables into symmetric matrix variables, we obtain a tight SDP relaxation for the proposed model. Our model is also different from the model of El Ghaoui et al. (2003), which considered the worst-case VaR robust portfolio problem, a special case of downside risk. Besides, El Ghaoui et al. solve the worst-case VaR model using second order cone programming (SOCP) instead of semidefinite programming (SDP) as is done in our paper. Numerical experiments and comparisons based on Shanghai Stock Exchange 50 index (SSE50) are presented and discussed in detail to test the performance of the proposed model. The numerical results indicate that the proposed robust tracking portfolio can track well and outperform SSE50 in many cases. Comparisons with the variance tracking error portfolio model and the equally weighted strategy indicate that our model's performance is more stable for different market data. In particular, we obtain much better Sharpe ratio in the investment period.

In the rest of the paper, unless otherwise specified, we will use the bold lowercase, e.g. $\mathbf{a}, \boldsymbol{\mu}, \ldots$, to denote a vector, and use N to denote the number of risky assets in benchmark. The other uppercase letters, e.g. $A, B, \Sigma, \ldots$, will generally denote a matrix. Conventional symbols such as $\mathbb{R}^n, \mathbb{S}^n$ and $\mathbb{S}^n_+$ are used to express respectively the spaces of n dimensional real vector, n dimensional square matrix and n dimensional positive semidefinite matrix. The relationship of $A-B \in \mathbb{S}^n_+$ is denoted by $A \succcurlyeq B$ or $A-B \succcurlyeq 0$. The inner product of two matrices A, B is denoted by $A \cdot B = \text{trace}(AB) = \sum_{i,j}^n a_{ij} b_{ij}$.

This paper is organized as follows. In Section 2, we define the worst-case downside risk measure and describe the robust tracking error portfolio problem. SDP formulations of the worst-case downside risk measure and the robust tracking error portfolio model are explored in Section 3. We report our numerical experiments and comparisons based on real market data in Section 4. Section 5 includes conclusion and some technical details are given in the Appendix.

2. Worst-case downside risk and robust tracking model

Let r_b be the random return of the tracked index (benchmark) consisting of N risky assets. Consider a tracking portfolio consisting of n risky assets. Denote the returns vector of n risky assets by $\mathbf{r} = (r_1, \ldots, r_n)^T \in \mathbb{R}^n$, where r_i is the gross return of the ith risky asset. Then the tracking error of returns between the tracking portfolio and the benchmark is

$$\Delta R = r_b - \sum_{i=1}^n r_i w_i$$

where $\mathbf{w} = (w_1, \ldots, w_n)^T \in \mathbb{R}^n$ is the weights vector of the tracking portfolio with w_i as the proportion invested in the ith risky asset and satisfies the budget constraint $\sum_{i=1}^n w_i = 1$. Let μ_b and $\boldsymbol{\mu}^r$ be the expected returns of the benchmark and n risky assets, (i.e. $\mathbb{E}[r_b] = \mu_b$ and $\mathbb{E}[\mathbf{r}] = \boldsymbol{\mu}^r$), respectively. Let the covariance matrix between r_b and $\mathbf{r}$ be

$$G = \begin{pmatrix} \sigma_b^2 & \mathbf{g}^T \\ \mathbf{g} & \Sigma \end{pmatrix},$$

where σ_b is the standard variance of the benchmark, $\mathbf{g}$ is the covariance vector between the n risky assets and the benchmark, and Σ is the covariance matrix of the n risky assets. Assume that (I) $n < N$ and (II) matrices G and Σ are positive definite. Assumption (I) means that we use fewer assets to track the benchmark. Assumption (I) in some sense is also