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Mean-downside-risk and mean-variance newsvendor models: Implications for sustainable fashion retailing

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ABSTRACT

Newsvendor models have been well-established for studying supply chain management problems with fashionable products. In this paper, we explore the mean-downside-risk (MDR) and mean-variance (MV) newsvendor models under both the exogenous and endogenous retail price decision cases. We first construct analytical models with the MDR and MV objectives. We then show that the analytical solution schemes for both the MDR and MV problems are the same. With the measures for sustainability such as the expected quantity of goods leftover, the expected sales to expected goods leftover ratio, the rate of return on investment, and the probability of achieving a pre-determined profit target, we proceed to compare the levels of sustainability by the fashion retailers which employ the mean-risk and the risk neutral models. Insights are generated.

1. Introduction

The newsvendor problem is a classical inventory management model which was first studied in the literature in the 18th century (Petruzzi and Dada, 1999). Owing to its intuitive and analytically tractable structure, the newsvendor problem has been widely applied in the studies of supply chain management problems with fashionable products. In particular, it is argued that the newsvendor problem can be used to model inventory decisions for fast fashion retailers such as Zara, H&M, Mango and Top Shop because of the close fitness between the newsvendor model and these companies' industrial practices of (i) high demand uncertainty, (ii) short and single selling season without no replenishment, and (iii) relatively simple cost-revenue structure with seldom markdowns (because of low inventory level and quick inventory turnover). The original model focuses on determining the optimal stocking quantity in a single-period single-item inventory problem with a stochastic demand. The objective function focuses on either expected cost minimization or expected profit maximization. Under this original model, it has been well-known that a unique optimal fractile quantity exists (Khouja, 1999).

However, in real world practices, companies usually would have different kinds of objectives and some decision makers are risk averse.[1] As a result, two streams of research emerged. The first stream focuses on determining the optimal stocking quantity for the newsvendor problem with the objective of optimizing the chance to achieve a target profit level (e.g., Lau, 1980; Sankarasubramanian and Kumaraswamy, 1983; Parlar and Weng, 2003). The second stream of research considers the optimal stocking decision when the newsvendor is risk averse. In order to capture the newsvendor's risk aversion, some researchers employ the utility function approach (e.g., Anvari, 1987; Chung, 1990; Eeckhoudt et al., 1995). However, the utility function approach is always being criticized by being impractical because of the tremendous difficulty in determining the utility function. As a result, some researchers employ the mean-risk models (e.g., the mean-variance (MV) model) for studying the risk averse newsvendor problem. Established by Markowitz in the 1950s, (Markowitz, 1959) the MV formulation has been widely applied for exploring financial investment problems with risk. Under the MV framework, an optimal decision is one which optimizes the problem with consideration of both payoff and risk where payoff is captured by the expected profit (mean) and risk is measured by the variance of profit. In the literature of newsvendor problem, Lau (1980) is the first piece of work which studies the newsvendor problem with an MV objective. After that, Chen and Federgruen (2000) study the newsvendor problem with a quadratic utility function and develop the MV non-inferior curve. They explore both the profit and cost oriented cases under the MV framework and make a comparison between them. Other works which employ the MV framework in studying the "newsvendor problem based" supply chain management problems include the study of supply chain returns policy under an MV framework in Lau and Lau (1999), the analysis of the commitment-option contracts with risk consideration in Buzacott et al. (2003), the study of both risk-averse

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[1] For the perspectives of risk management in supply chains which affect substantially the optimal decisions in the companies, refer to Tang (2006).

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and risk-prone MV newsvendors in Choi et al. (2008a), and others (Choi et al., 2008b; Wei and Choi, 2010; Wu et al., 2009). Despite the intuitive meaning behind the MV model, there is a serious inherent theoretical flaw associated with it. Essentially, the MV model captures the level of risk by using the variance of profit. Since risk is only associated with the uncertain unfavourable outcomes but variance includes both the uncertain upside (favourable) and downside (unfavourable) deviations from the mean, unless the downside and upside deviations are symmetric,[2] the use of variance for capturing risk has flaws. This is the case for the newsvendor problem because the distribution of the profit function is in general asymmetric (even if the market demand follows a symmetric distribution). As a consequence, there are proposals to use a downside risk measure to replace variance and establish the mean-downside-risk model. For instance, Gan et al. (2004) have used the mean-downside risk (MDR) measure in studying supply chain coordination problem with risk averse newsvendors.

Based on the above literature, we first propose to model a fashion retailer's inventory decision making problem as a newsvendor problem. We then develop the respective optimization problems with both the MV and the MDR objectives for the exogenous and endogenous retail pricing cases, respectively. We show that the optimal solutions for both models (under both the exogenous and endogenous retail pricing schemes) are the same. On the other hand, sustainability and green operations management have been popular topics in production economics (Bonney and Jaber, 2010; Sundarakani et al., 2010; Vachon and Klassen, 2008; Yang et al., 2010). It is of course related to the fashion retailing problem and it will be interesting to explore how different objective functions affect the retailers' decisions as well as the respective level of sustainability. As a result, in this paper, we propose to measure the sustainability of the fashion retailer from various perspectives which include the expected amount of leftover (environmental friendliness), the expected sales to expected goods leftover ratio (environmental friendliness and economic sustainability), the rate of return on investment (economic sustainability), and the probability of meeting the profit target (economic sustainability). Insights are generated.

2. Exogenous retail pricing case

2.1. Mathematical model

We first examine the case when the retail price is exogenously given and hence is not a decision variable. We consider a fashion retailer which operates following the newsvendor problem (Lau 1980). To be specific, the fashion retailer orders a certain amount of fashionable product from the manufacturer with a wholesale price c. The product is sold in the market with a unit selling price p. For the sake of brevity and without loss of insights, we consider the case that any unsold product becomes obsolete and is hence wasted. The product's demand is denoted by x and we model it as a random variable following a probability density function $f(x)$ and a cumulative distribution function $F(x)$. We assume that the moments of x are finite. We represent $F^{-1}(\cdot)$ as the inverse function of $F(\cdot)$ and there is a one to one mapping between $F(\cdot)$ and its argument. Before the season starts, the fashion retailer needs to determine the stocking quantity q. Obviously, unless the retailer orders nothing (it is a trivial case) or else we have $q \geq 1$. For this fashion retailing inventory problem, for a given q, the fashion retailer's profit is given below

$$\pi(q) = (p-c)q - p(q-x)^+,$$

where $(q-x)^+ = \max(q-x, 0)$.

Taking expectation and variance with respect to x, we have the fashion retailer's expected profit and variance of profit as follows:

$$E[\pi(q)] = (p-c)q - p\int_0^q F(x)dx,$$

$$V[\pi(q)] = p^2\left(2q\int_0^q F(x)dx - 2\int_0^q xF(x)dx - \left[\int_0^q F(x)dx\right]^2\right).$$

Define $\sigma[\pi(q)]$ as the standard deviation of profit, i.e.,

$$\sigma[\pi(q)] = \sqrt{V[\pi(q)]}.$$

For the downside risk measure, we employ the semi-deviation which is defined as follows:

$$S[\pi(q)] = E\{(E[\pi(q)] - \pi(q))^+\} = p\int_0^{q-\int_0^q F(x)dx}\left[q - \int_0^q F(x)dx - x\right]f(x)dx. \quad (2.1)$$

The structural properties of $E[\pi(q)]$, $V[\pi(q)]$, and $S[\pi(q)]$ are summarized in Proposition 2.1 below.

Proposition 2.1. *(a) $E[\pi(q)]$ is a concave function. (b) $\sigma[\pi(q)]$ is an increasing function. (c) $S[\pi(q)]$ is an increasing function.*

Proof of Proposition 2.1: All proofs are placed in Appendix (A1).

Based on Proposition 2.1, we can see that: (i) there exists an optimal stocking quantity q_{E^*} which maximizes $E[\pi(q)]$ and is given by

$$q_{E^*} = \arg_q\{dE[\pi(q)]/dq = 0\} = F^{-1}[(p-c)/p], \quad (2.2)$$

(ii) $q = 0$ minimizes both $V[\pi(q)]$ *and* $S[\pi(q)]$.

Proposition 2.2. $\sigma[\pi(q)] \geq S[\pi(q)]$.

Proposition 2.2 is intuitive as $S[\pi(q)]$ only captures the downside deviation from the mean whereas $\sigma[\pi(q)]$ captures both upside and downside deviations from the mean.

2.2. Mean-downside-risk and mean-variance models

With the structural properties of $E[\pi(q)]$, $V[\pi(q)]$, and $S[\pi(q)]$, we can define the following mean-downside-risk (MDR) and mean-variance (MV) models for the case when the retail price is exogenous[3]:

$$(\mathrm{P_{MDR}}) \quad \min_q S[\pi(q)] \quad \text{subject to } E[\pi(q)] \geq \alpha,$$

$$(\mathrm{P_{MV}}) \quad \min_q \sigma[\pi(q)] \quad \text{subject to } E[\pi(q)] \geq \alpha,$$

where α is the expected profit target that the fashion retailer would like to achieve.

To have meaningful models, α must be bounded above by the maximum achievable expected profit, i.e.,

$$\alpha \leq E[\pi(q_{E^*})]. \quad (2.3)$$

Define:

$$q_{\alpha^*} = \arg_{q \leq q_{E^*}}\{E[\pi(q)] = \alpha\}. \quad (2.4)$$

[2] The symmetric case, such as the asset's price is normally distributed, is usually assumed for a direct application of the MV model in financial portfolio optimization problems.

[3] Notice that there are alternative model formulations for the MV and MDR problems. Please refer to Appendix (A2) for some discussions.