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Pricing the term structure of inflation risk premia: Theory and evidence from TIPS

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ABSTRACT

In this paper, we study inflation risk and the term structure of inflation risk premia in the United States' nominal interest rates through the Treasury Inflation Protection Securities (TIPS) with a multi-factor, modified quadratic term structure model with correlated real and inflation rates. We derive closed form solutions to the real and nominal term structures of interest rates that drastically facilitate the estimation of model parameters and improve the accuracy of the valuation of nominal rates and TIPS prices. In addition, we contribute to the literature by estimating the term structure of inflation risk premia implied from the TIPS market. The empirical evidence using data from the period of January 1998 through October 2007 indicates that the expected inflation rate, contrary to data derived from the consumer price indices, is very stable and the inflation risk premia exhibit a positive term structure.

1. Introduction

Inflation always plays an essential role in our economy. Having experienced the high inflation era due to the oil crisis in the 1970s, many countries have since introduced inflation-protected securities. For investors who rely on the stability and predictability of fixed income investing finding ways to limit or mitigate the effects of inflation is crucial for continued financial security. The fear of inflation has also made hedging inflation risk a top priority in the investment world. Because of the surge of investors' demand, the inflation-indexed security market has been growing quickly.[1] Inflation-linked securities have been widely accepted by many institutional and individual investors.[2] Treasury Inflation Protection Securities (TIPS) in the United States were first introduced in January 1997 and expanded quickly. As of October 2007, a total of 25 TIPS have been traded (including two expired TIPS) in the market, and the TIPS market has grown to over $310 billion, or 6.3% of the total outstanding Treasury debt by the end of 2007.

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[1] Inflation protected securities are also issued by private entities. FHLMC (Freddie Mac), Tennessee Valley Authority Power, Household Finance, and John Hancock Life Insurance Company issue corporate bonds that are also indexed to CPI-U (unlike TIPS where the principal amount is adjusted, these inflation protected corporate notes feature an adjustment of the coupon rate). CPI indexed annuities are also available from Irish Life Company of North America (ILONA) and TIAA-CREF. Inflation protected CDs that are FDIC insured are also available.

[2] For example, pension funds use the inflation-linked market as a natural match for their long-dated liabilities; life insurance companies desire a hedge against policies linked to inflation; and investors, such as large endowments, use TIPS as a new asset class to provide portfolio diversification.

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Although TIPS were only introduced in the U.S. in 1997, inflation-indexed bonds have a long history. The world's first known inflation-indexed bonds were issued by the Commonwealth of Massachusetts in 1780 during the Revolutionary War, when those bonds were invented to deal with severe wartime inflation and with anger among soldiers in the U.S. army with the decline in purchasing power of their pay.[3] While the appearance of the first inflation-indexed bond dates back to 1780, inflation-indexed bonds did not flourish until the twentieth century.

The TIPS traded in the U.S. have an inflation-adjusted principal and a fixed coupon rate. Coupon payments are inflation-protected as a result of the inflation-adjusted principal. The index used for determining the inflation adjustment is the non-seasonally adjusted Consumer Price Index-Urban Consumer (CPI-U), published monthly by the U.S. Department of Labor. Even though the CPI-U index is published monthly, a daily interpolation algorithm is used from two adjacent month's index values to calculate daily accrued interests in the secondary market.[4] Since the interest payments of the TIPS are "inflation-protected,"[5] the difference between yields of the TIPS and yields of the corresponding nominal bonds is usually perceived as the expected inflation. However, this crude approximation ignores the inflation risk premium and neglects the fact that inflation and real interest rates are highly correlated.

In this paper, we derive and test a two-factor term structure model for the inflation rate and the real rate. We assume that the first factor explains the prices of TIPS completely and the sum of the two factors explains the nominal interest rates that are derived from nominal CMT (Constant Maturity Treasury) rates. Theoretically, our model is similar to Richard's model (1978) where the nominal rate is approximated by the Fisher equation. However, the model presented here differs from the Richard model in that the "inflation factor" is not inflation itself. Rather, it is a factor that follows inflation closely. This model also differs from the Richard model due to incorporation of correlation between the factors. Technically, our model can be regarded as a special case of the Duffie and Kan (1996) model, which is in turn a special case of the Duffie-Pan-Singleton model (2000). The Duffie-Pan-Singleton model solves a rich class of payoff functions for fixed income derivatives that include the payoffs of default-free coupon and zero coupon bonds. By concentrating on this special case of the Duffie-Pan-Singleton model, we are able to provide true closed form solutions as opposed to ones that require solving a series of ODEs (Ordinary Differential Equations).

Empirically, our model directly compares to Jarrow and Yildirim (2003) who price TIPS and ordinary Treasuries with the HJM model. However, due to the limitation of the HJM model, they cannot estimate the inflation risk premium. Moreover, the correlation between the real rate and inflation is computed separately and hence is inconsistent with the estimation of the other parameters. In contrast, in our model, all parameters, including the correlation, are estimated consistently within the model.

The paper is organized as follows. Section 2 coves a brief literature review of recent term structure models and inflation. In Section 3, we derive a true closed form solution that requires no ODE solution to the modified quadratic term structure model where the real interest rate and inflation are correlated. In Section 4, the technique used to estimate the model with TIPS and nominal CMT rates is discussed. The empirical results are described in Sections 6 summarizes our conclusions.

2. Related literature

This paper is motivated by two strands of literature on inflation. One, known as the Fisher approximation, is the literature on the nominal term structure where the nominal rate is approximated by the sum of the real rate and expected inflation. This is a "reduced form" approach to model inflation. The other literature takes the "structural" approach where inflation is endogenously determined in a monetary economy. Using the continuous time methodology for the first time, Cox, Ingersoll and Ross (CIR, 1985) unite the two approaches and derive a multi-factor general equilibrium model for the nominal bond in which an exact relationship between the real rate, inflation, and the nominal rate is defined and the Fisher equation is validated as a first order approximation.

Following CIR, models for the nominal term structure have grown in three directions. The first set of models assumes multiple factors that explain nominal interest rates, but, do not explicitly assume the factors to be the real rate and inflation. They instead leave the factors unspecified. These include, among many others, (i) Langetieg (1980), Hull and White (1990), Turnbull and Milne (1991), Longstaff and Schwartz (1992), Duffie and Kan (1996), Dai and Singleton (2000), and Duffie et al. (2000) under the affine formulation, (ii) Inui and Kijima (1998) under the HJM formulation, and (iii) Constantinides (1992), Ahn, Dittmar and Gallant (2002), Leippold and Wu (2002), Kim (2004), and Ang, Boivin and Dong (2008) under the quadratic formulation. Our model can be regarded as a special case of the Duffie-Kan, Dai-Singleton or Duffie-Pan-Singleton model. However, it provides us a true closed-form solution (as opposed to a series of ordinary differential equations) which improves efficiency and accuracy in the empirical study.

The second approach adopts the Fisher approximation directly and decomposes the nominal rate into the real rate, inflation, and sometimes the inflation risk premium.[6] These models all assume independence between the real rate and inflation in order to gain mathematical tractability. However, unlike the previous approach where factors are left unspecified, it is very hard to justify the assumption of independence. Our model solves this problem by explicitly incorporating the correlation between the real rate and inflation in the model while retaining closed-form solutions.

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[3] Interested readers may refer to Shiller (2003) for more historical information on inflation-indexed bonds.

[4] The following link from the Treasury Department provides an example of this daily adjustment: http://www.treasurydirect.gov/instit/annceresult/tipscpi/2009/cpi_20091118.pdf. In order to calculate daily Index (interpolation) adjustments, TIPS require index levels from two prior months. Since the publication of the Index is usually lagged by one month, in order to calculate, for example, December 2009 index ratio, we have to use September 2009 and October 2009 data.

[5] The exact details of the protection will be discussed later.

[6] Note that the instantaneous nominal rate is the sum of the instantaneous real rate and the instantaneous inflation rate. There is no inflation risk premium. Risk premia exist for term rates.