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Fading memory learning in a class of forward-looking models with an application to hyperinflation dynamics

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Abstract

We analyze a class of non-linear deterministic forward-looking economic models (the state today is affected by today's and tomorrow's expectation) under bounded rationality learning. The learning mechanism proposed in this paper defines the expected state as a geometric average of past observations. We show that the memory of the learning process plays a stabilizing role: it enlarges the local stability parameters region of the perfect foresight stationary equilibria and it eliminates non-perfect foresight cycles–attractors generated through local bifurcations. In a hyperinflation economy with two stationary equilibria we show that only one of the two equilibria can be stable under bounded rationality learning provided that agents have a 'long memory'. We can also have convergence towards non-perfect foresight cycles and even chaotic dynamics. If the debt financing quota is small enough then no restriction on the agent's memory is needed. The equilibrium with the higher inflation rate is stable under bounded rationality for a small (and not really significant) set of economies. © 2001 Elsevier Science B.V. All rights reserved.

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1. Introduction

The aim of this paper is twofold. To analyze a class of non-linear deterministic forward-looking models under bounded rationality learning with fading memory and to analyze a hyperinflation economy assuming that the agents employ this type of learning mechanism.

The class of economic models analyzed in this paper is characterized by the fact that the state today is affected by today's and tomorrow's agent expectation. We investigate this class of models assuming that the agents are not fully rational, i.e. they do not know the complete economic model and they employ a recursive learning mechanism to update their beliefs. We consider a learning mechanism characterized by *fading memory*, i.e. the agents evaluate the time $t+1$ expectation as a weighted average of the values of the state observed up to time $t-1$. The weights of the average are described by a geometric progression with a ratio smaller than 1 and therefore, the weights for older observations are smaller than the weights for recent observations. Different from other learning mechanisms proposed in the literature, the learning step of our mechanism does not go to zero as time goes on. The model with learning is characterized by the stationary equilibria obtained under perfect foresight, whereas the other attractors (cycles, invariant sets, complex attractors) are not of perfect foresight. Asymptotically, the dynamics of the agents' expectations are fully described by the first order auto-regressive learning scheme with a constant learning step. The analysis of this type of learning scheme gives us information about the attractors of the original learning scheme, but no information can be obtained about their basin of attractions.

The learning mechanism proposed in this paper is similar to the one analyzed in Bray (1982, 1983), Evans and Honkapohja (1994, 1995), where the agent's expectation is computed as the arithmetic average of past observations. In that framework there is full memory (the same weight is employed for each observation) and rewriting the agents' expectations recursively with a time varying learning step the learning step goes to zero as time goes on (vanishing learning step). In our framework there is not full memory, remote observations are less relevant than recent observations. Memory is inversely related to the learning step which does not go to zero as time goes on. These features of our learning mechanism are appealing because the vanishing of the learning step and the assumption of a constant weight for past observations are not fully plausible from a behavioral point of view. As a matter of fact, agents do not stop to learn as time goes on and they 'forget' remote observations.

In our analysis we show that the memory of the learning process plays a stabilizing role in a local sense. As memory becomes longer, we observe two interesting phenomena: the region of local stability of the stationary perfect foresight equilibria (SPFE) are enlarged and non-perfect foresight attractors (cycles–invariant sets) generated through local bifurcations disappear. Simulations show that memory dampens the fluctuations but, it does not necessarily simplify the nature of the fluctuations. This type of result confirms the findings obtained in