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# Endogenizing the provision of money: Costs of commodity and fiat monies in relation to the value of trade

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ABSTRACT

We consider the problem of including the costs and value of the institutions that define money and support trade, within the framework of economic optimization. We compare monetary systems mediated by durable commodity monies, versus pure fiat monies, in order to understand the separation and eventual independence of the institutionally-created value of money from the values of underlying traded goods. We treat the emergence of monetary function as a problem in mechanism design, modeled by minimal strategic market games that overcome a generalized Jevons failure when agents must commit ahead of time to specialist resource production. We consider in particular the problem of defining closures with respect to both money flows and labor-allocation and trading decisions, and show that minimal models require many of the fundamental institutions of banking and contract enforcement found in real economies, in order to define a self-policing system. We define costs, value, and the efficiencies of the institutions that support trade in terms of a natural money-metric welfare function, and compare the characteristics of commodity and fiat monies by these measures. Through careful treatment of the stock/flow distinction in repeated-game settings, we find that commodity money, even when its value derives heavily from its institutional role, remains defined by its flow characteristics, in contrast to fiat money, for which the control function is defined inherently in terms of stock variables. Our notation is somewhat nonconventional for economics but to do justice to econo-physics concepts such as scaling and dimensional analysis and to stress the distinction between stocks and flows, we believe this notation is justified. We provide a full listing of notation in Appendix A.



## 1. Introduction

Money and the institutions that support trade must be represented in two ways in economic models. They are the framework that enables and controls economic activity, but they are also the output of productive work that competes for resources with other activities internal to the economy.

This dual role presents a formal problem for economic theory when considering commodity monies, which exist both as pre-institutional trade goods, and as the carriers of shadow prices reflecting their institutional roles as store of value or medium of exchange. Commodity monies do, however, retain some of their pre-institutional character, as their value in trade is naturally anchored in production and consumption. The more a money's function is institutionally determined, the weaker this anchor becomes and the less it explains. For the idealization of a pure fiat money, the pre-institutional and static optimization problem recognized by Economic General Equilibrium theory entirely omits the economic institutional sector from its models of production enabling trade and consumption.

The factors affecting the choice of money systems, the size of the monetary sector in terms of both function and cost, and the limits to allocative efficiency that depend on money, arise from the value of trade and the production technologies for institutions. In the current essay we present a way to embed these factors within closed, minimal models of economic optimization.

### 1.1. Adding money to the pre-institutional society and to General Equilibrium theory

The entity that is actually modeled by General Equilibrium theory is the pre-institutional society. It consists of identified goods, constraints on their production, and utilities from their consumption. Economic activity is reflected abstractly in the

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[1] We adopt the convention that in joint work the order of appearance of names on the publication should be selected randomly unless there is a specific stated reason otherwise. We have acted accordingly.

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analysis of optimizing allocations, but the economic process itself is not represented by elements within models. In General Equilibrium theory, both the pre-institutional society and the concepts of optima are static. In order to understand not only money but dynamics, the equilibrium analysis must be replaced by fully-defined games of strategy.

We use a standard equilibrium model of the pre-institutional society as a setting within which trade can only occur when its mechanisms are fully specified, and we then compare minimal mechanisms that use a durable commodity money and a government-produced fiat money. The optimizing allocations of General Equilibrium, which also arise as limits of dynamically explicit solutions, then provide a reference for allocative efficiency. We study both the regularization of exchange prices for goods that are already within the pre-institutional model, and the "price" of the monetary system determined by its marginal cost and the marginal value of trade.

#### *1.1.1. Prior work: the regularization of trading patterns and the stabilization of prices through commodity exchange*

The regularization of trading patterns and convergence of prices may readily be treated as solutions to problems of bargaining and search with commodity exchange (Kiyotaki and Wright, 1989; Rubinstein and Wolinsky, 1985; Starr, 1976; Gale, 1986; Bak et al., 2001), but the resulting "money" remains principally a good *within* the economy. These solutions belong to abstract binary trade theory and are pre-institutional in orientation. They can be used experimentally and in some instances furnish a justification for economic anthropology. Without institutional support, any stable money price must be closely linked to consumer valuation of the underlying good, and properties of the "monetary sector" such as money supply and velocity have limited freedom to differ from those of other non-money goods in the economy (in a sense that we make precise below).

#### *1.1.2. Four other classes of formal approach to monetary theory*

Monetary models overall are distinguished from General Equilibrium in that the latter is usually presented without money and without institutional or strategic context, whereas the former require some version of a full process description. In addition to binary trade models, we distinguish four other categories of models.

Class 1 comprises the "old macroeconomics", including Keynes, Tobin, Friedman and many others which is openly applied and institutionally oriented. These are bolstered by more mathematical and less ad hoc institutional characterizations in works of Hicks, Patinkin, Gurley and Shaw. In Class 2 we put the new classical economics espoused primarily by Lucas and associates (Lucas, 1980) who apply techniques of parallel dynamic programming to macroeconomic policy questions. We believe that these models may reflect minimal institutions, but lack sufficient dimensionality to be of direct value to policy. Parallel dynamic programming has been used similarly to model money and minimal institutions in the macroeconomy by (Karatzas et al., 1994). What we may call Class 3 contains works of Diamond and Dybvig, Wallace, Corbai, Temzelides and Wright, Kehoe, Levine and Woodford and others. These do not fit easily into the above categories, and their relations are well described in the survey essay of Wallace (2001) covering monetary economics and banking from a different point of view.

As class 4 we take the approaches we have termed "mathematical institutional economics" (Shubik, 1959). Their foundation is the conversion of pre-institutional and static General Equilibrium models into *strategic market games* (Shubik, 1999). The emphasis in their design is that the mechanisms required to assure well-defined play under all conditions correspond to minimal models of essential institutions.

### *1.2. The problem of closures and minimal models*

General Equilibrium represents a *minimal* theory in that it seeks to derive prices entirely from abstractions of competition, marginal productivity, and marginal utility of consumption (including consumption of services). We have argued (Smith and Shubik, 2005) that minimal models for the explicit dynamics of one-period trade may be similarly abstracted, but that they require classification according to (1) numbers of strategic degrees of freedom made available to agents, (2) the constraints that limit strategic action, and (3) symmetries. In this essay we introduce related models and criteria of minimality for the provision of monies and the monetary functions that require multiple interacting timescales.

Monetary models require closure, both of the flows of traded goods from production to consumption, and of money. Because they capture dynamics, the role and economic meaning of initial and final conditions must also be specified.

Closure and boundary conditions for commodity money are given as a modest extension of the solutions for one-period trade in Smith and Shubik (2005), by introducing an extensive-form game with repeated stages of trading via the one-period models, augmented by carry–forward of money. The only new problem is solution of the Hahn paradox for salvage value in the limit of infinite durability. We present a simplified and somewhat unrealistic solution of this problem in the usual terms of perfect information, and then interpret this solution in more realistic terms that include the role of uncertainty. These solutions are minimal by the same criteria used for one-period models.

To model fiat money we must introduce several new institutions to create and distribute money, and to handle default. Many of these, in the one-period setting, have been modeled with respect to the role of bank lending and variable objectives of the central bank in Kannai and Rosenmüller (2010). As a model-closure device for the multi-period problem, we link the costs of the monetary system, and the value of money in trade, to competition for labor in the enforcement of debt contracts and the collection of taxes, along the basic lines suggested by Knapp (1905). We may characterize the objective of government, implemented through joint policies of the central bank and of taxation, as stabilizing trade at the most efficient level permitted by trading-post models, without strategic default by any class of agents in the model.

It is more difficult, in the multi-institution models we introduce for fiat money, to define formal criteria by which our abstractions are minimal idealizations, than it was for one-period exchange or for durable commodity money. However, we argue that the fundamental asymmetry that distinguishes the government from other actors in the economy is the ability to exercise coercion, placing this actor outside the limits of fully voluntary exchange. Following from this asymmetry, all the other features in our extensive-form game model are required by the control functions of fiat money, or by its inherent independence from consumable goods. The following list explains why classes of institutions are needed:

1. In a minimal model where perfect trust or cost-free clearinghouses are not assumed, standardization of fiat money is achieved by its centralized production. It is not automatically available to all agents, and a stable money supply cannot simply be assumed to reflect "history" as could be done for a commodity money, the supply of which is anchored in production and consumption. Fiat money therefore requires a central bank to distribute it in a manner compatible with its value in trade, in a way that a commodity money does not.