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Coalition-proof Nash allocation in a barter game with multiple indivisible goods☆

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Abstract

We consider a barter market with multiple types of indivisible goods. Konishi et al. [J. Math. Econ. 35 (2001) 1–15] showed that the core of this market may be empty. We define a normal form game for this market and consider its equilibrium outcomes. Assuming separable strict preferences, we show that the game has a unique Coalition-proof Nash allocation. This Coalition-proof Nash allocation is precisely the unique commodity-wise competitive allocation of the market, and it may be inefficient. The commodity-wise competitive allocation is implementable in Coalition-proof Nash equilibria but not in Strong Nash equilibria.

Keywords: Coalition-proof Nash equilibrium; Commodity-wise competitive allocation; Indivisible goods; Separable preferences; Shapley–Scarf economy

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1. Introduction

To study the effects of indivisibilities in market economies, Shapley and Scarf (1974) presented their celebrated "house barter economy". In this model, there are a finite number

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of players, each endowed with a single indivisible item, e.g., a house.[1] The houses are *differentiated*, meaning that players may value the various houses in the market differently. However, no divisible good exists in this market. Furthermore, the individual players have no use for more than one house. Hence, they will try to trade houses among themselves in some mutually beneficial way.

In spite of its discreteness, this economy turned out to have nice properties. First, the core is always nonempty (Shapley and Scarf, 1974). If the players have strict preference orderings, then a unique competitive allocation exists and it coincides with the strict core, a subset of the core (Roth and Postlewaite, 1977). In addition, the strict core allocation rule is the only social choice function with Pareto optimality, individual rationality and strategy-proofness (Ma, 1994). Moulin (1995) suggested that a market economy in which multiple types of indivisible goods are exchanged should be considered to see if the distinctive properties of the house barter economy can be generalized. This paper pursues this direction of research.

We consider the following market. There are n players. They trade m types of indivisible goods, say, houses, cars, computers, etc. Each player is endowed with m items, one unit of each type of good. Thus, there are n indivisible items of each type in the market. All the items of each type are differentiated. For each type of good, each player wants to consume exactly one unit. He has a preference ordering over all bundles (m-tuples) consisting of one item of each type. By exchanging some of his own items for others of the same types, each player tries to consume a more preferred m-tuple. We emphasize that there is no divisible good in this market. The players simply permute their ownerships of items.

The market we consider is an extension of the house barter economy, which is the case with m=1. The first research on this extended market was done by Konishi, Quint and Wako (2001). KQW (2001) showed that this extension has quite different properties from the house barter economy. In particular, the core may be empty even if all individual preferences are strict and *separable*.[2] Hence, we need to take another approach to see what outcome can be attained as an equilibrium.

In this paper, we define a normal form game for the market. In the game, a strategy of each player is to demand one item of each type. For each type, if his demand along with others' demands form a *trading cycle*, then he gets the item he demands. Otherwise, he keeps his initial item. Using this game, we analyze how each player forms trading cycles to obtain a preferable bundle of items. To see an equilibrium outcome of this trading cycle formation, we examine Strong Nash and Coalition-proof Nash equilibria of the game. While a Strong Nash equilibrium is immune to any coalitional deviation, a Coalition-proof Nash equilibrium is immune to any *self-enforcing* coalitional deviation. Thus, even if no Strong Nash equilibrium exists, we may still be able to analyze a Coalition-proof Nash equilibrium.

[1]The model is called the "house barter economy", since a house is a typical example of an indivisible good that can saturate one's preferences with only one unit.

[2]We say that the preferences of a player are *separable* if his preferences between two items of the same type, e.g., two houses, do not depend upon which items of other types he consumes.