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Multi-unit differential auction–barter model for electronic marketplaces

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ABSTRACT

Differential auction–barter (DAB) model augments the well-known double auction (DA) model with barter bids so that besides the usual purchase and sale activities, bidders can also carry out direct bartering of items. The DAB model also provides a mechanism for making or receiving a differential money payment as part of the direct bartering of items, hence, allowing bartering of different valued items. In this paper, we propose an extension to the DAB model, called the multi-unit differential auction–barter (MUDAB) model for e-marketplaces in which multiple instances of commodities are exchanged. Furthermore, a more powerful and flexible bidding language is designed which allows bidders to express their complex preferences of purchase, sell and exchange requests, and hence increases the allocative efficiency of the market compared to the DAB. The winner determination problem of the MUDAB model is formally defined, and a fast polynomial-time network flow based algorithm is proposed for solving the problem. The fast performance of the algorithm is also demonstrated on various test cases containing up to one million bids. Thus, the proposed model can be used in large-scale online auctions without worrying about the running times of the solver.

1. Introduction

The *double auction* (DA) institution allows market participants to submit asks (sale bids) and bids (purchase bids) for well-defined commodities, financial instruments or services. Beginning with Smith (1962), many laboratory experiments with DA rules have been carried out (see Davis and Holt (1993), Holt (2006) for a survey). The experiments demonstrate that the DA institution results in high allocative efficiency even with a small number of traders (Smith 1982, Plott 1982, Friedman 1984) which also explains the wide usage of the DA institution in the exchanges. The popularity of the institution has motivated researchers to propose and study variants of the DA. The *single-unit DA* is the base model in which each commodity in the market is considered as unique item. The *multi-unit DA* (Plott and Gray 1990, Huang et al. 2002) is an extension to the single unit DA which allows multiple instances of a commodity to be traded in the auction. Kalagnanam et al. (2001) introduce multi-unit DA with assignment constraints and propose a network flow algorithm for finding the optimum allocation of commodities. In the (*multi-unit*) *combinatorial double auction* (Fan et al. 1999, Xia et al. 2005), the institution further allows package (combinatorial) bidding in which the participants can submit bids on bundles of items instead of a single item. For an introductory survey of the DA, the reader is referred to (Friedman 1993).

An *electronic marketplace* (*e-marketplace*) is an electronic exchange that brings buyers and sellers together providing necessary regulations and services for trading. E-marketplaces offer services such as directory listings and searching of goods or services and transaction support. Secure, fast and reliable communication mediums in e-marketplaces help their customers in making new trading partnerships and reducing communication costs. However, for increasing the trading volume in the e-marketplaces, besides these services, a matching/brokering service is necessary between sellers and buyers in order to satisfy their possibly complex supply and demand requirements (Dumas et al. 2004). Auction based approaches have been proposed for this task (Ono et al. 2003). For instance, Özturan (2005) proposes a hybrid differential auction–barter (DAB) model which extends the single-unit DA institution for increasing the trading volume of online used vehicle auctions such as autobytel.com, the eBay Motors and the Yahoo! Autos. In these auctions, buyers place bids for the vehicles they want to purchase and sellers place asks for the cars they want to sell. Although bidders can submit both sale and purchase bids, these bids are considered as independent and they are not allowed to put conditional restrictions for their bids. This discourages bidders who are willing to sell their current vehicles only if they are able to purchase other vehicles. For instance, assume that a bidder currently owns a car of brand *A* and wants to exchange (barter) his car for a car of brand *B* while paying at most $ 5000. He must first determine reservation prices for both car *A* and car *B*. The *reservation price* is the minimum (maximum) price at which the bidder is willing to sell (buy) an item. Assuming that the reservation price

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of the bidder for his car *A* is \$ 10,000, he will place an ask of \$ 10,000 for his car *A* and a bid of \$ 10,000 + \$ 5000 = \$ 15,000 for car *B*. Depending on the auction outcome, he may lose his car without getting a replacement car or be forced to pay \$ 15,000 for an additional vehicle without selling his current vehicle, resulting in a \$ 10,000 budget deficit. In order to prevent this inefficiency, the DAB model extends the single-unit DA so that the bidders are also allowed to place *barter bids* in which an item can be bartered for another item. Besides, if the values of the items are not considered to be same, then the bidders are also allowed to declare an amount of money, called *differential price*, to be given or taken along with their barter request. For instance, for the above case, the bidder can submit a bid declaring that he wants to exchange his car *A* for car *B* and pay \$ 5000. This bid is represented using the following notation:

$$\{\text{car } A + \$\ 5000\} \Rightarrow \{\text{car } B\}$$

Since the DAB model is designed especially for online used car auctions (consumer-to-consumer), it is very unlikely to have more than one unit of an item (e.g. two identical used cars). Therefore, it is designed as a single-unit auction model and does not support multiple identical units of an item. The bidding language is also quite simple. A barter bid consists of exactly two items, an item to be given and an item to be taken, and an associated differential price if any. Similarly, a sale bid or a purchase bid consists of a single item and an associated price the bidder is willing to pay or earn.

In this paper, we propose a new model called the *multi-unit differential auction–barter* (MUDAB) model for e-markets in which multiple instances of commodities are exchanged. The model has been primarily designed for improving the efficiency of a clearinghouse (Friedman 1993) which is also known as call-market (Satterthwaite and Williams 1993) or the discrete-time DA institution. In a *clearinghouse*, traders submits asks and bids during a predefined trading period, and at the end of this period, the market is cleared by matching the asks with the bids.

The model has three important features which are also explained quantitatively based on an example market scenario in the next section:

(i) The model extends both the multi-unit DA and the DAB models so that bidders can put forward barter bids along with the sale and purchase bids for multiple instances of items. This mechanism encourages traders to participate in the e-markets without risking buying of new items unless they sell their items first or vice-versa. Furthermore, in addition to the direct matching of sale and purchase bids as in DA, the model increases the allocative efficiency of the market by extracting barter cycles and sale–barter–purchase chains of any length in the market. An example scenario for a barter cycle involving three bidders is as follows:

$$\underbrace{\text{Bidder 1}}_{\textit{Barter}} \xrightarrow{\text{gives item(s)to}} \underbrace{\text{Bidder 2}}_{\textit{Barter}} \xrightarrow{\text{gives item(s)to}} \underbrace{\text{Bidder 3}}_{\textit{Barter}} \xrightarrow{\text{gives item(s)to}} \text{Bidder 1}$$

An example scenario for sale–barter–purchase chain involving four bidders is given below:

$$\underbrace{\begin{array}{c}\text{Bidder 1}\\ \text{(earns money)}\end{array}}_{\textit{Sale}} \xrightarrow{\text{gives item(s)to}} \underbrace{\text{Bidder2}}_{\textit{Barter}} \xrightarrow{\text{gives item(s)to}} \underbrace{\text{Bidder3}}_{\textit{Barter}} \xrightarrow{\text{gives item(s)to}} \underbrace{\begin{array}{c}\text{Bidder 4}\\ \text{(pays money)}\end{array}}_{\textit{Purchase}}$$

Finally, since the MUDAB model is the superset of both the multi-unit DA and the DAB models, the allocative efficiency of the MUDAB model is guaranteed to be at least as good as that of the multi-unit DA and the DAB models.

(ii) The model introduces a powerful and flexible bidding language (which is explained in Section 4 in detail) which allows bidders to express their complex preferences inside bids. Compared to DAB, the new bidding language allows bidders to combine purchase, sell and exchange requests inside the bids and put restrictions on the number of items to be traded. This feature improves the allocative efficiency of market, and also makes the MUDAB model preferable over the DAB model even for single-unit markets. Furthermore, the new bidding language reduces the total number of bids required for representing preferences of bidders greatly, and hence, reduces the amount of online information exchange between the bidders and the auctioneer.

(iii) The optimum allocation in the MUDAB model, that is the optimum set of satisfiable bids, can be determined using the proposed fast polynomial-time algorithm which is based on network flows (Ahuja et al. 1993). Thus, the model can be used in large-scale web-based auctions containing millions of bids without worrying about the running times of the solver.

In the rest of the paper, we first compare the DA, DAB and MUDAB institutions quantitatively with an example scenario. Section 3 presents some electronic commerce applications that our model could be applied. In Section 4, a more comprehensive example with which we explain our MUDAB model is given. In Section 5, we formulate our model by using linear-integer programming and define the winner determination problem. In Section 6, we introduce a minimum cost network flow solution of the winner determination problem. Section 7 presents the experimental results and finally, the paper is concluded in Section 8.

2. Comparison of DA, DAB and MUDAB models

In this section, the DA, DAB and MUDAB models will be compared based on an example scenario from a business-to-business paper market involving 5 bidders. The scenario is presented in Table 1a. In the paper industry, paper is manufactured in different qualities which we denote as grade *A*, *B* or *C* and is traded in standard sized rolls (Kalagnanam et al. 2001). In this scenario, Bidder 1 wants to sell 200 rolls of his inventory, and Bidders 4 and 5 want to purchase 100 rolls of grade *A* and *B* paper respectively. Bidders 2 and 3, on the other hand, want to exchange (barter) their 100 rolls of inventory for lower grade paper. The reason could be that lower grade paper could satisfy their requirements and they would like to earn money by exchanging their inventory. The reservation prices of the bidders for different grades of paper are presented in Table 1b.

If the market was regulated using single-unit or multi-unit DA rules, then the first bidder would want to sell 200 rolls of grade *B* paper. The reason is that he considers the price of grade *B* paper higher than the price of grade *C* paper, and therefore, he would want to make more profit. Since there is no bartering option available in DA, the second and the third bidders either would not participate in the market because of the risk of losing their inventory and the risk of having a budget deficit, or they would take these risks and submit sale and purchase bids based on their subjective reservation prices. No decision making would be necessary for the fourth and the fifth bidders, and hence, they would simply submit purchase bids for the paper they want. As shown in Table 1c, the outcomes of the market would be same for both the single-unit and multi-unit cases. If the second and the third bidders do not participate, then only the sale bid of the first bidder would match the fifth bidder's purchase bid, causing a total of 100 rolls of paper