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Examining the evidence of purchasing power parity by recursive mean adjustment[☆]

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ABSTRACT

This paper revisits the empirical evidence of purchasing power parity under the current float by recursive mean adjustment (RMA) proposed by So and Shin (1999). We first report superior power of the RMA-based unit root test in *finite samples* relative to the conventional augmented Dickey–Fuller (ADF) test via Monte Carlo experiments for 16 linear and nonlinear autoregressive data generating processes. We find that the more powerful RMA-based unit root test rejects the null hypothesis of a unit root for 16 out of 20 current float real exchange rates relative to the US dollar, while the ADF test rejects only 5 at the 10% significance level. We also find that the computationally simple RMA-based asymptotic confidence interval can provide useful information regarding the half-life of the real exchange rate.

1. Introduction

Purchasing power parity (PPP) asserts that the real exchange rate is a mean reverting stochastic process around its long-run equilibrium level. PPP serves as a key building block for many open economy macro models. Despite its popularity and extensive research, empirical validity of PPP remains inconclusive due to the mixed empirical evidence.

Testing for long-run PPP is typically carried out by implementing unit root tests for real exchange rates. Studies employing conventional augmented Dickey–Fuller (ADF) tests find very little evidence of PPP with the current float (post Bretton Woods system) real exchange rates. It is well known that the ADF test has low power when the time span of the data is relatively short. Indeed, empirical studies that use long-horizon data, rather than using the current float data, find more favorable evidence for PPP (Taylor, 2002a, among others).[2] In an effort to overcome the power problem, an array of research employed panel unit-root tests for the current float data and report evidence in favor of PPP. It should be noted, however, that (first-generation) panel unit-root tests may be oversized (Phillips and Sul, 2003).[3] Therefore, it is not clear that panel approaches using the current float data solve the power problem.

Another important issue we note is the following. It is a well-known statistical fact that the least squares (LS) estimator for autoregressive (AR) processes suffers from serious small-sample bias when the stochastic process includes a non-zero intercept and/or deterministic time trend. The bias can be substantial especially when the process is highly persistent (Andrews, 1993).

Since the pioneering work of Kendall (1954), many bias-correction methods have been developed. Andrews (1993) proposed a method to obtain the exactly median-unbiased estimator for AR (Andrews, 1993) process with normal errors. Andrews and Chen (1994) extend the work of Andrews (1993) and develop the approximately median-unbiased estimator for AR(p) processes. Hansen (1999) developed a nonparametric bias correction method of grid bootstrap that is robust to distributional assumptions.

Murray and Papell (2002) employ methods proposed by Andrews (1993) and Andrews and Chen (1994) to correct for the downward median-bias in the persistence parameter estimates and find that confidence intervals for the half-lives of most current float real exchange rates extend to positive infinity. Based on this, they conclude that the univariate estimation methods provide no useful information on the real exchange rate dynamics. Similar evidence is reported by Rossi (2005).

We revisit these issues by employing an alternative method, recursive mean adjustment (RMA) by So and Shin (1999), that belongs to a class of (approximately) mean-unbiased estimators. The RMA estimator is computationally convenient to implement yet powerful and has been employed in various studies. For instance, Choi et al. (2010) develop an RMA-based bias-reduction method for dynamic panel data models. Sul et al. (2005) employ RMA to mitigate prewhitening bias in heteroskedasticity and autocorrelation consistent estimation. Taylor (2002b) employs RMA for a seasonal unit root test and found superior size and

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[2] See Rogoff (1996) for a survey.
[3] Phillips and Sul (2003) show that conventional panel unit-root tests tend to reject the null of unit root too often in presence of cross-section dependence. O'Connell (1998) finds much weaker evidence for PPP controlling for cross-section dependence.

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power properties. Cook (2002) applied RMA to correct a severe oversize problem of the Dicky–Fuller test in the presence of level break. Kim et al. (2010) compare RMA with Hansen's (1999) grid bootstrap method for estimating the half-life of international relative equity prices.

We first demonstrate superior *finite sample* performance (in terms of power) of the RMA-based unit root test over the ADF test by Monte Carlo experiments for 16 linear and nonlinear autoregressive data generating processes.[4] We also show that, unlike the LS-based methods, a simple RMA asymptotic confidence interval can provide good coverage properties.[5]

To evaluate its practical usefulness, we test the null hypothesis of unit root for 20 current float quarterly real exchange rates relative to the US dollar. We note that the more powerful RMA-based unit root test rejects the null for 16 countries while the conventional ADF test rejects the null only for 5 countries at the 10% significance level. Second, unlike Murray and Papell (2002) and Rossi (2005), we obtain compact confidence intervals for the half-lives for those countries that pass the RMA-based unit root test.

The remainder of the paper is organized as follows. Section 2 describes So and Shin's (1999) RMA and three alternative methods to construct confidence intervals for the persistence parameter estimate. In Section 3, we present Monte Carlo simulation results to evaluate the finite sample performance of the unit root test with RMA. Section 4 reports our main empirical results with the current float exchange rate data. Concluding remarks follow in the last section.

2. The methodology

2.1. Recursive mean adjustment

Let p_t be the domestic price level, p_t^* be the foreign price level, and e_t be the nominal exchange rate as the unit price of the foreign currency in terms of the home currency. All variables are expressed in natural logarithms and are integrated processes of order 1. When PPP holds, there exists a cointegrating vector $[1 \quad 1 -1]'$ for the vector $[e_t \; p_t^* \; p_t]'$, the log real exchange rate, $s_t = e_t + p_t^* - p_t$, can be represented by a stationary AR process such as,

$$s_t = c + u_t, \qquad u_t = \sum_{j=1}^{p} \rho_j u_{t-j} + \varepsilon_t, \tag{1}$$

where $\rho = \sum_{j=1}^{p} \rho_j$ is less than one in absolute value ($|\rho| < 1$) and ε_t is a mean-zero white noise process. Equivalently, the AR model (1) can be alternatively represented by,

$$s_t = c(1-\rho) + \sum_{j=1}^{p} \rho_j s_{t-j} + \varepsilon_t, \tag{2}$$

which implies the following augmented Dickey–Fuller form,

$$s_t = (1-\rho)c + \rho s_{t-1} + \sum_{j=1}^{k} \beta_j \Delta s_{t-j} + \varepsilon_t, \tag{3}$$

where $k = p-1$, $\beta_j = -\sum_{s=j+1}^{p} \rho_s$, and $\rho = \sum_{j=1}^{p} \rho_j$ as previously defined.

Assuming that PPP holds, the persistence parameter ρ can be estimated by the conventional LS estimator. When $p = 1$, Eq. (1) can be written as,

$$s_t = (1-\rho)c + \rho s_{t-1} + \varepsilon_t \tag{4}$$

By the Frisch–Waugh–Lovell theorem, Eq. (4) can be equivalently estimated by,

$$s_t - \bar{s} = \rho(s_{t-1} - \bar{s}) + \eta_t, \tag{5}$$

where $\bar{s} = T^{-1} \sum_{i=1}^{T} s_i$ is a sample mean and $\eta_t = \varepsilon_t - (1-\rho)c - (1-\rho)\bar{s}$. Note that ε_t, and thus η_t, is correlated with the demeaned regressor $(s_{t-1} - \bar{s})$ because ε_t is correlated with s_i for $i = t, t+1, \cdots, T$, which is embedded in the regressor $(s_{t-1} - \bar{s})$ through $\bar{s}$. Since the exogeneity assumption fails, the LS estimator, $\hat{\rho}_{LS}$, is biased. The bias has an analytical representation and one can obtain the exactly mean-unbiased estimate by using a formula developed by Kendall (1954).[6]

This paper corrects for the bias by employing an alternative method, the recursive mean adjustment (RMA), proposed by So and Shin (1999). The RMA method is computationally simple yet powerful and flexible enough to deal with higher order AR models. For this, rewrite Eq. (4) as,

$$s_t - \bar{s}_{t-1} = \rho(s_{t-1} - \bar{s}_{t-1}) + \xi_t, \tag{6}$$

where $\bar{s}_{t-1} = (t-1)^{-1} \sum_{i=1}^{t-1} s_i$ is the recursive mean and $\xi_t = \varepsilon_t - (1-\rho)c - (1-\rho)\bar{s}_{t-1}$. Since ε_t is orthogonal to the adjusted regressor $(s_{t-1} - \bar{s}_{t-1})$, the RMA estimator $\hat{\rho}_{RMA}$ substantially reduces the bias.

When $p = k+1 > 2$, we follow a single-equation version of Choi et al.'s (2010) method. That is, we first estimate Eq. (3) by the LS and construct the following.

$$s_t^+ = (1-\rho)c + \rho s_{t-1} + \varepsilon_t^+, \tag{7}$$

where $s_t^+ = s_t - \sum_{j=1}^{k} \hat{\rho}_{j,LS} \Delta s_{t-j}$ and $\varepsilon_t^+ = \varepsilon_t - \sum_{j=1}^{k} \left(\hat{\rho}_{j,LS} - \rho_j\right) \Delta s_{t-j}$. Then, we apply RMA to (Hall, 1994),

$$s_t^+ - \bar{s}_{t-1} = \rho(s_{t-1} - \bar{s}_{t-1}) + \nu_t, \tag{8}$$

where $\nu_t = \varepsilon_t^+ + (1-\rho)c - (1-\rho)\bar{s}_{t-1}$. Finally, the RMA estimator $\hat{\rho}_{RMA}$ is obtained by,

$$\hat{\rho}_{RMA} = \frac{\sum_{i=2}^{T} (s_{t-1} - \bar{s}_{t-1})(s_t^+ - \bar{s}_{t-1})}{\sum_{i=2}^{T} (s_{t-1} - \bar{s}_{t-1})^2} \tag{9}$$

After estimating $\hat{\rho}_{RMA}$ and its associated standard error, one can use the RMA-based ADF *t*-statistic to test the null hypothesis of a unit-root ($H_0: \rho = 1$). As shown by Shin and So (2001), the RMA-based unit root test possesses greater power asymptotically than the LS-based ADF unit root test. Due to reduced-bias estimation, the left *p*th percentile of the null distribution of the test statistic shifts to the right, while asymptotic distributions of the RMA and LS estimators are identical under the alternative. This leads to an improvement in power over the LS-based unit root test.

[4] Shin and So (2001) show that the RMA-based unit root test is asymptotically more powerful than the LS-based test. This is because reduced-bias estimation of the RMA method shifts critical values (distribution under the null hypothesis) to the right, while the RMA and LS estimators share the same asymptotic distributions under the stationary alternative hypothesis, which therefore results in power improvement.

[5] As Murray and Papell (2002) note, there is no clear reason to prefer the median unbiased estimator to mean-bias correction methods for general autoregressive models, even though both approaches are successful in reducing each type of bias (Kilian, 1998, 1999). However, we present some evidence that RMA-based methods have some desirable properties when one wants to test an economic hypothesis that implies stationarity of a certain variable. In what follows, we demonstrate that the RMA-based ADF test has superior power over the LS-ADF test for 16 linear and nonlinear stationary stochastic processes. Furthermore, while the median-unbiased method tends to generate very wide half-life confidence intervals that often extends to positive infinity, the RMA method tends to provide compact confidence intervals when the stationarity is supported. Since we investigate PPP that requires stationary real exchange rates, we believe that RMA-based methods are more useful because the RMA-ADF test is more powerful than the LS-ADF test and RMA estimator can help draw practically meaningful inferences on the persistence of PPP shocks.

[6] Tanaka (1984) and Shaman and Stine (1988) extend Kendall's exact mean-bias correction method to AR(p) models. However, their methods are computationally complicated when the lag order is large.