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Threshold-autoregressive, median-unbiased, and cointegration tests of purchasing power parity

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Abstract

We use Dickey-Fuller tests, threshold autoregressive unit-root tests, median unbiased estimators, and cointegration tests for $I(1)$ and $I(2)$ variables to examine the validity of Purchasing Power Parity (PPP). The within-sample tests generally lead to the rejection of long-run PPP. Long-term out-of-sample forecasts assuming various forms of long-run PPP are not especially better than those assuming that real rates contain a unit-root. We show that no one method emerges as the ''best'' in the sense that it provides the smallest out-of-sample forecast errors. © 1998 Elsevier Science B.V.

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1. Introduction

The central premise of Purchasing Power Parity (PPP) is that the rate of foreign currency appreciation should equal the inflation differential between the domestic and foreign country. Certainly, nations with high inflation rates do have depreciating currencies. Moreover, the terms ''overvalued'' and ''undervalued'' typically make reference to a currency's PPP level. Despite the theory's intuitive appeal, there is still widespread disagreement on the matter of mean reversion in real exchange rates. For example, Adler and Lehman (1983); Enders (1988); Mark (1990) show that the real exchange rates of industrialized nations exhibit large fluctuations with inordinately slow rates of decay. If there is any mean reversion, the point estimates for the decay factors average close to 2% per month; the implication is that any deviation from PPP has a half-life approximating three years.

A possible explanation for the empirical shortcomings of PPP concerns the low power of standard Dickey and Fuller (1979) unit-root tests to discriminate between non-stationary versus near-stationary processes. Given the estimated rates of convergence in the real exchange rate series, it is not surprising that there is controversy concerning the issue of whether real rates are slow-decaying or unit-root processes. At first sight, the difference between slow-decaying and unit-root processes might not seem important. However, if a real exchange rate has a unit-root, deviations from PPP are permanent such that the expected rate of foreign currency appreciation is not equal to the inflation differential between the domestic and foreign country.

One aim of this paper is to invoke a number of newly developed econometric methods that can potentially discriminate between unit-root and near unit-root behavior in the real exchange rate series of

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the industrialized nations. Specifically, we use Dickey and Fuller (1979) tests, Enders and Granger's (Enders and Granger, 1997) threshold autoregressive (TAR) unit-root tests, Fuller's (Fuller, 1995) median unbiased estimators, and Johansen's (Johansen, 1995) cointegration tests for $I(1)$ and $I(2)$ variables. The appropriate use of each methodology is described and applied to the various real exchange rate series. The second aim of the paper is to compare the various methods concerning their ability to generate accurate forecasts. We show that no one method emerges as the ''best'' in the sense that it provides the smallest out-of-sample forecast errors. Overall, the results are not particularly favorable to the PPP hypothesis. The within-sample tests generally lead to the rejection of long-run PPP. Long-term out-of-sample forecasts assuming various forms of long-run PPP are not especially better than those assuming that real rates contain a unit-root.

2. Tests for unit roots

To illustrate the issues involved, consider the following econometric model of Purchasing Power Parity:

$$p_t^* + e_t - p_t = r_t \tag{1}$$

where: p_t^* = logarithm of the index of the foreign price level in t; e_t = logarithm of the domestic currency price of foreign exchange in period t relative to a base year; p_t = logarithm of the index of the domestic price level in t; and r_t is a stochastic disturbance representing a deviation from PPP. Note that r_t is typically called the ''real'' exchange rate or ''real price'' of foreign exchange.

The rationale for writing p_t^*, e_t and p_t as left-hand-side variables is to emphasize the fact that all three are jointly determined. Viewed in this way, PPP is not a ''Theory of The Exchange Rate'' any more than it is a ''Theory of Differential Inflation Rates.'' The long-run version of PPP implies that r_t is stationary. After all, if r_t is not stationary, deviations from PPP contain a permanent component such that any discrepancy from PPP is never fully eliminated.

Using monthly data from the CD-ROM version of the International Monetary Fund's *International Financial Statistics* we obtained Consumer Price Index (CPI) and nominal exchange rate data for Belguim, Canada, France, Germany, Greece, Italy, Japan, Luxembourg, the Netherlands, Spain, Switzerland, the United Kingdom, and the United States. The data covers the period from January 1973 through December 1996. Alternately, using the United States, Germany, and the United Kingdom as the ''foreign'' country, we constructed real exchange rates in accord with Eq. (1). The four panels of Fig. 1 show the real U.S. exchange rate with each of the other twelve countries through December 1993. (As explained later, three years of data are withheld in order to perform out-of-sample forecasts.) In general, the real dollar fell throughout the 1970's, rose until the mid-1980's and then began a decline towards its 1970's level. From the figure, it is clear that deviations from PPP are large and persistent. We focus on the issue of whether or not there is actually mean reversion present in the data. Also note that, all of the U.S. real rates with the European nations seem to move in tandem. We also address the issue of whether or not PPP performs better for the European nations than for the U.S.

In order to test whether the real exchange rates are stationary, we performed unit-root tests on each of the series using the following procedure:

Step 1: Using the U.S., Germany, and the U.K. as the base country, we constructed real exchange rates as indicated by Eq. (1) and applied OLS to estimate regression equations of the form:

$$\Delta r_t = \alpha_0 + \rho r_{t-1} + \alpha_1 \Delta r_{t-1} + \alpha_2 \Delta r_{t-2} + \ldots + \alpha_k \Delta r_{t-k} + \epsilon_t \tag{2}$$

Note that a deterministic trend is not included in Eq. (2) since the presence of a trend is inconsistent with long-run PPP. We retained three years of data in order to perform out-of-sample forecasts and estimated the regression over the sample period 1973:1–1993:12. The key feature to note in Eq. (2) is the value of ρ. If $-2<\rho<0$, the real exchange rate sequence will revert to a long-run mean. However, if it is not possible to reject the null hypothesis $\rho=0$, the $\{r_t\}$ sequence will not be stationary.

Step 2: One problem in implementing eq. (2) is the determination of the lag length k. We selected lag lengths using the Bayesian Information Criterion