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Optimality of barter steady states[a]

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Abstract

In the present paper, the issue of the existence of optimal barter steady states is addressed in a stationary overlapping generations model with many commodities and many different long-lived consumers. A general characterization of the individual endowments leading to an optimal barter steady state is obtained. The result, which is a generalization of Benveniste and Cass (1986), is obtained as a corollary of an unpublished result of Cass (1982). © 1998 Elsevier Science B.V. All rights reserved.

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1. Introduction

Barter steady states in which aggregate savings are zero can be obtained by competitive markets, without requiring any institution such as money or a central clearing house. In this regard, the issue of the existence of an optimal barter steady state is relevant from an economical point of view (see, e.g., Samuelson (1958) or Gale (1973)).

In the present paper we consider a stationary overlapping generations model with many commodities and many different long-lived consumers. In this

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framework, a complete characterization of individual initial endowments leading to optimal barter steady states as well as non-optimal barter steady states is obtained. The characterization, which is a direct consequence of an unpublished result of Cass (1982), shows that when the endowment distribution is sufficiently asymmetrical in favor of the latest year of consumer's life, no 'government' is necessary to reach optimality.

The present issue has been addressed in an economy with a representative consumer living two periods by Benveniste and Cass (1986). In economies with long-lived consumers results have been obtained by Grandmont (1983) and Wang (1993) under restrictive assumptions on the characteristics of the consumers, mainly the discounted form of the utility function and the monotonicity of savings as a function of the interest rate.

2. The model

A double-ended stationary overlapping-generations model with no production is considered. In every period, there are l commodities and a generation of m consumers is born. The consumers live and have endowments for n periods.

Consumer i is described by his consumption set $X_i = \prod_{s=1}^{n} Y_i$, endowment, $\omega_i = (\omega_i^s)_{s=1}^n$, and a utility function u_i. The economy is described by the consumers in one generation, because all generations are identical, $\mathscr{E} = (X_i, \omega_i, (u_i)_{i=1}^m)$. The usual assumptions hold: $Y_i = \mathbb{R}^l_{++}$, $\omega_i^s \in \mathbb{R}^l_{++}$, u_i is $C^\infty(Y_i, \mathbb{R})$, strictly quasi-concave, has strictly positive derivatives and the indifference surfaces are contained in the positive orthant. A consumer i in generation t maximizes his utility function subject to his budget constraint:

$$\max\, u_i(x_{it}^1, \ldots, x_{it}^n)$$

$$\text{s.t. } q(t) \cdot x_{it} = q(t) \cdot \omega_i,$$

where $x_{it} = (x_{it}^1, \ldots, x_{it}^n) \in \mathbb{R}^{nl}_{++}$ is the consumption bundle and $q(t) = (p(t), \ldots, p(t+n-1)) \in \mathbb{R}^{nl}_{++}$ the price vector for periods t to $t+n-1$. For consumer i, utility maximization leads to the demand function f_i: $\mathbb{R}^{nl}_{++} \times \mathbb{R}_{++} \to \mathbb{R}^{nl}_{++}$. The aggregate excess demand function of generation t in the sth period of life is defined as

$$y^s(q(t), \omega) = \sum_{i=1}^{m} f_i^s(q(t), q(t) \cdot \omega_i) - \sum_{i=1}^{m} \omega_i^s,$$

where $s \in \{1, \ldots, n\}$ and $\omega = (\omega_i)_{i=1}^m \in \Omega(r) = \{\omega \in \mathbb{R}^l_{++}, \sum_{i=1}^m \sum_{s=1}^n \omega_i^s = r\}$.

Definition 1. An *equilibrium* $((p(t))_{t\in\mathbb{Z}}, \omega)$ is a sequence of prices and endowments such that $p(t) \in \mathbb{R}^l_{++}$ and markets clear:

$$\sum_{s=1}^{n} y^s(q(t-s+1), \omega) = 0$$