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A slacks-based measure of super-efficiency in data envelopment analysis: An alternative approach

Hsin-Hsiung Fang [a,b], Hsuan-Shih Lee [b,*], Shiuh-Nan Hwang [c], Cheng-Chi Chung [b]

[a] Department of Air and Sea Logistics and Marketing, Taipei college of Maritime Technology, Taiwan
[b] Department of Shipping and Transportation Management, National Taiwan Ocean University, Keelung, Taiwan
[c] School of Management, Ming Chuan University, Taipei, Taiwan

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ABSTRACT

The current paper proposes a slack-based version of the Super SBM, which is an alternative super-efficiency model for the SBM proposed by Tone. Our two-stage approach provides the same super-efficiency score as that obtained by the Super SBM model when the evaluated DMU is efficient and yields the same efficiency score as that obtained by the SBM model when the evaluated DMU is inefficient. The projection identified by the Super SBM model may not be strongly Pareto efficient; however, the projection identified from our approach is strongly Pareto efficient.

1. Introduction

Since the advent of data envelopment analysis (DEA), which was first introduced by Charnes et al. [2], many papers have been published on its methodology and applications. There are two types of DEA models, the radial and non-radial models. The CCR model measures the radial efficiency of the inputs (input-oriented) or outputs (output-oriented) by gauging the ratio of the inputs to be contracted or the ratio of the outputs to be enlarged so that the evaluated DMU becomes efficient. One of the limitations of radial models is that radial efficiency does not reflect all inefficiency of a DMU [12]. Slacks need to be considered simultaneously with radial efficiency to identify the "real" projection of a DMU. To overcome this, Charnes et al. [3] developed additive model of DEA, which deals with input excesses and output shortfalls directly. Though the additive model can discriminate between efficient and inefficient DMUs, the model provides no efficiency measure so that decision maker can tell how well a DMU performs.

In light of these issues, Tone [14] proposed a non-radial model called SBM (slacks-based measure), which uses the term "slacks" to represent the input excesses and output shortfalls and deals with them directly and by maximizing theses slacks. The hallmark of SBM is that SBM provides efficiency score which is units-invariant and a monotone function of input slacks and output slacks.

To break the ties of efficient DMUs, Andersen and Petersen [1] proposed a radial super-efficiency model under the condition of constant returns to scale (CRS). The super-efficiency model under the condition of variable returns to scales (VRS) may suffer from infeasibility. Chen et al. [6] proposed a modified VRS super-efficiency model which successfully addresses the infeasibility issues occurring either in conventional VRS models or the N–L superefficiency model. Chen [4] proposed a modified model to tackle the infeasibility occurred when super-efficiency data envelopment analysis is used in ranking the efficient DMUs. For more detail discussions on the infeasibility, please refer to Seiford and Zhu [13], Lovell and Rouse [11], Cook et al. [8], Chen [5], Lee et al. [9], Chen and Liang [7], and Lee and Zhu [10].

As a non-radial approach, Tone [15], based on SBM, proposed another model to rank efficient DMUs. Tone's Super SBM requires that standard SBM is run first to classify efficient and inefficient DMUs, and next Super SBM is run only for the efficient DMUs. However, the projection identified by Super SBM may not be strongly Pareto efficient. In this paper, we propose an alternative two-stage approach so that the projection identified will be strongly Pareto efficient and the efficiency score is the same as Tone's approach. We transform Tone's Super SBM into a slack-based version so that identified slacks can be incorporated into the standard SBM. With such modification, the slack-based version of Super SBM and the revised SBM can work collaboratively. We reverse the sequence of optimizations, where the slack-based version of Super SBM is run first and then the revised SBM is run to determine the real projection and standard SBM score. This paper is organized as follows: Section 2 briefly reviews the SBM model and the Super SBM model. In Section 3, the alternative

* Corresponding author. Tel.: +886 9 18401038; fax: +886 2 24631903.
E-mail addresses: hslee@mail.ntou.edu.tw, hslee@ntou.edu.tw (H.-S. Lee).

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approach is presented. Numerical examples are demonstrated in Section 4. Some remarks will follow in Section 5.

2. Preliminaries

Suppose there are n DMUs associated with m inputs and s outputs. Let x_{ij} denote the ith input of DMU j and y_{rj} denote rth output of DMU j. Assume that all data are positive, i.e., $x_{ij}, y_{rj} > 0$ for all possible $i=1,\ldots,m$; $r=1,\ldots,s$; $j=1,\ldots,n$.

The production possibility set P spanned by all DMUs is defined as

$$P = \left\{ (x_1,\ldots,x_m,y_1,\ldots,y_s) \,|\, x_i \ge \sum_{j=1}^{n} \lambda_j x_{ij}, i=1,\ldots,m, y_r \le \sum_{j=1}^{n} \lambda_j y_{rj}, r=1,\ldots,s \right\} \tag{1}$$

Tone [14] proposed the following SBM model to evaluate the efficiency of DMU k

$$\begin{aligned} \min \rho = & \frac{1-(1/m)\sum_{i=1}^{m} z_i^-/x_{ik}}{1+(1/s)\sum_{r=1}^{s} z_r^+/y_{rk}} \\ s.t.\quad & x_{ik} = \sum_{j=1}^{n} x_{ij}\lambda_j + z_i^-, i=1,\ldots,m \\ & y_{rk} = \sum_{j=1}^{n} y_{rj}\lambda_j - z_r^+, r=1,\ldots,s \\ & \lambda_j \ge 0, j=1,\ldots,n \\ & z_i^- \ge 0, i=1,\ldots,m \\ & z_r^+ \ge 0, r=1,\ldots,s \end{aligned} \tag{2}$$

Tone [14] defines that a DMU is SBM-efficient if $z_i^{-*} = z_r^{+*} = 0$ for all i and r. Or equivalently, a DMU is SBM-efficient if $\rho^* = 1$.

For a SBM-efficient DMU k, Tone [15] proposed the following model (Super SBM) to identify its super-efficiency:

$$\begin{aligned} \min \delta = & \frac{(1/m)\sum_{i=1}^{m} \bar{x}_i/x_{ik}}{(1/s)\sum_{r=1}^{s} \bar{y}_r/y_{rk}} \\ s.t.\quad & \bar{x}_i \ge \sum_{j=1, j\ne k}^{n} x_{ij}\lambda_j, i=1,\ldots,m \\ & \bar{y}_r \le \sum_{j=1, j\ne k}^{n} y_{rj}\lambda_j, r=1,\ldots,s \\ & \lambda_j \ge 0, j=1,\ldots,n, j\ne k \\ & \bar{x}_i \ge x_{ik}, i=1,\ldots,m \\ & \bar{y}_r \ge 0, \bar{y}_r \le y_{rk}, r=1,\ldots,s \end{aligned} \tag{3}$$

If we employ model (3) to evaluate an inefficient DMU, the efficiency will be 1. In other words, inefficient DMUs cannot be discriminated by Super SBM.

Model (3) does not incorporate slacks explicitly. The following model presents a slacks-based representation of Super SBM, whose representation is more consistent with model (2):

$$\begin{aligned} \min \delta = & \frac{1+(1/m)\sum_{i=1}^{m} w_i^-/x_{ik}}{1-(1/s)\sum_{r=1}^{s} w_r^+/y_{rk}} \\ s.t.\quad & x_{ik} \ge \sum_{j=1, j\ne k}^{n} x_{ij}\lambda_j - w_i^-, i=1,\ldots,m \\ & y_{rk} \le \sum_{j=1, j\ne k}^{n} y_{rj}\lambda_j + w_r^+, r=1,\ldots,s \\ & \lambda_j \ge 0, j=1,\ldots,n, j\ne k \\ & w_i^- \ge 0, i=1,\ldots,m \\ & w_r^+ \ge 0, w_r^+ \le y_{rk}, r=1,\ldots,s \end{aligned} \tag{4}$$

Theorem 1. *Model (4) and model (3) are equivalent.*

Proof. Substituting $\bar{x}_{ik}$ with $x_{ik}+w_i^-$ and $\bar{y}_{rk}$ with $y_{rk}-w_r^+$, we have

$$\begin{aligned} \min \delta = & \frac{(1/m)\sum_{i=1}^{m} (x_{ik}+w_i^-/x_{ik})}{(1/s)\sum_{r=1}^{s} (y_{rk}-w_r^+/y_{rk})} \\ s.t. & x_{ik}+w_i^- \ge \sum_{j=1, j\ne k}^{n} \lambda_j x_{ij}, i=1,\ldots,m \\ & y_{rk}-w_r^+ \le \sum_{j=1, j\ne k}^{n} \lambda_j y_{rj}, r=1,\ldots,s \\ & x_{ik}+w_i^- \ge x_{ik}, i=1,\ldots,m \\ & y_{rk}-w_r^+ \le y_{rk}, y_{rk}-w_r^+ \ge 0, r=1,\ldots,s \\ & \lambda_j \ge 0, j=1,\ldots,n; j\ne k \end{aligned}$$

After rearrangement, we have

$$\begin{aligned} \min \delta = & \frac{(1/m)\sum_{i=1}^{m} (x_{ik}+w_i^-/x_{ik})}{(1/s)\sum_{r=1}^{s} (y_{rk}-w_r^+/y_{rk})} = \frac{1+(1/m)\sum_{i=1}^{m} (w_i^-/x_{ik})}{1-(1/s)\sum_{r=1}^{s} (w_r^+/y_{rk})} \\ s.t. & x_{ik}+w_i^- \ge \sum_{j=1, j\ne k}^{n} \lambda_j x_{ij}, i=1,\ldots,m \\ & y_{rk}-w_r^+ \le \sum_{j=1, j\ne k}^{n} \lambda_j y_{rj}, r=1,\ldots,s \\ & w_i^- \ge 0, i=1,\ldots,m \\ & w_r^+ \ge 0, w_r^+ \le y_{rk}, r=1,\ldots,s \\ & \lambda_j \ge 0, j=1,\ldots,n; j\ne k \end{aligned}$$

□.

Model (4) identifies the projection in the fourth quadrant of DMU k by minimizing the input savings (w_i^-) and output surpluses (w_r^+). It is worth noting that $w_r^+ \le y_{rk}$ is necessary to ensure that the objective function to be positive.

3. The alternative approach

Let (w_i^{-*}, w_r^{+*}) denote the optimal solution of (4). We revise the standard SBM model (2) as follows:

$$\begin{aligned} \min & \frac{1-(1/m)\sum_{i=1}^{m} (s_i^-/x_{ik})}{1+(1/s)\sum_{r=1}^{s} ()s_r^+/y_{rk}} \\ s.t.\quad & x_{ik} = \sum_{j=1, j\ne k}^{n} x_{ij}\lambda_j - w_i^{-*} + s_i^-, i=1,\ldots,m \\ & y_{rk} = \sum_{j=1, j\ne k}^{n} y_{rj}\lambda_j + w_r^{+*} - s_r^+, r=1,\ldots,s \\ & \lambda_j \ge 0, j=1,\ldots,n, j\ne k \\ & s_i^- \ge 0, i=1,\ldots,m \\ & s_r^+ \ge 0, r=1,\ldots,s \end{aligned} \tag{5}$$

Instead of solving (2) first and then applying (3) to the efficient DMUs, we reverse the sequence. Our approach is that model (4) is applied to all DMUs first and model (5) is applied so that inefficient DMUs can be discriminated.

The logic behind our approach is that if DMU k is outside the production possibility set spanned by DMUs excluding DMU k, (4) will first identify the minimum distance for DMU k from the frontier in terms of the input savings (w_i^-) and the output surpluses (w_r^+). By adding input saving to DMU k and subtracting output surpluses from DMU k, DMU k will be able to move to the frontier. However, so far, the projection identified might not be Pareto efficient. To remedy such problem, model (5) is employed to identify the possible input excesses (s_i^-) and output shortfalls (s_r^+). If DMU k is not SBM-efficient, i.e., DMU k is inside the production possibility set spanned by DMUs excluding DMU k,