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Quantifying the efficiency of price-only contracts in push supply chains over demand distributions of known supports

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ABSTRACT

In this paper, we quantify the efficiency of price-only contracts in supply chains with demand distributions by imposing prior knowledge only on the support, namely, those distributions with support $[a,b]$ for $0<a\le b<+\infty$. By characterizing the *price of anarchy* (PoA) under various push supply chain configurations, we enrich the application scope of the PoA concept in supply chain contracts along with complementary managerial insights. One of our major findings is that our quantitative analysis can identify scenarios where the price-only contract actually maintains its efficiency, namely, when the demand uncertainty, measured by the *relative range* b/a, is relatively low, entailing the price-only contract to be more attractive in this regard.

1. Introduction

Price of anarchy (PoA), a quantifier measuring the inefficiency of a multi-agent system due to selfish behavior of its agents, has been an extremely popular concept in computer science and operations research communities in the last decade Nisan et al. [17]. Perakis and Roels [19] pioneered its application in supply chain contracts and obtained the PoA for price-only contract for several configurations of the underlying supply chain when the demand distributions possess the (weakly) *increasing generalized failure rate* (IGFR) property. A non-negative random variable X with cumulative distribution function (cdf) $F(x)$ and probability density function (pdf) $f(x)$ is of IGFR property if $xf(x)/\overline{F}(x)$ is nondecreasing for all x such that $\overline{F}(x)>0$, where $\overline{F}(x)=1-F(x)$. We will use $\mathcal{F}_{\text{IGFR}}$ to denote the class of all distributions with IGFR property.

One of the most important managerial insights observed from their analysis is that the worst PoA under $\mathcal{F}_{\text{IGFR}}$ is at least 1.71 (a 71% loss of efficiency) even for the simple two-stage chain, and consequently price-only contract may not be a viable practical contract with certain demand distributions due to this large loss of efficiency. Nevertheless, the price-only contract has been widely adopted in many real-life practices. This popularity has been constantly attributed to its low administrative cost (cf. [1]).

Therefore, the following important questions arise naturally: is it possible that the assumption of IGFR on demand distributions leads to the overwhelmingly negative image on the price-only contract? Can we identify and justify those situations where price-only contract is attractive not just because of its low administrative cost?

These questions of significant practical consequences serve the main motivation of this work, which investigates another class $\mathcal{F}[a,b]$ $(0<a\le b<+\infty)$ of all distributions with support of the form $[a,b]$. The adoption of this class is a distributionally robust (or distribution-free, semi-parameter or min–max) approach similar to those by Scarf (1965) (see also, e.g., [8–13,21,20], etc.), when only distribution parameters, such as support, mean or variance, rather than the full distribution itself, are assumed to be known.

The main contribution of this work is to derive PoA bounds over all distributions in the class $\mathcal{F}[a,b]$ under various supply chain configurations, as compared with the work of Perakis and Roels [19] for distribution class $\mathcal{F}_{\text{IGFR}}$. These two classes of distributions are overlapping but not inclusive: there are IGFR distributions with support $[0,\infty)$ and there are distributions of support $[a,b]$ that are not IGFR. The bounds derived by Perakis and Roels [19] depend on the number of supply chain partners n and the profit margin, whereas the bounds derived here depend on the number of supply chain partners n and the relative range b/a. Hence, different and complementary managerial insights are obtained, especially with regard to the degree of uncertainty of demand, measured by the parameter b/a.

We only present the results on the *push* mode [2], where the downstream partner(s) hold(s) the supply chain inventory. Interested readers are referred to our working paper [6] for results concerning the other mode, pull mode. Moreover, throughout this

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paper, we will only consider pure equilibria for all the (sub-)games involved, as mixed strategies are not well-accepted in supply chain management [3]. Finally, we only focus on the nontrivial cases where the upstream partner in the game is the leader and where full efficiency cannot be achieved, that is, PoA > 1.

The paper is organized as follows. After this introduction, we first provide some preliminary results in Section 2, and then consider the serial supply chain system, the assembly system, and two distribution systems depending on two different customer behaviors in Section 3, Section 4, and Section 5, respectively. We conclude the paper by some important observations in Section 6.

All technical proofs can be found in the Appendices.

2. Preliminaries

2.1. The centralized setting

For any given supply chain system, we imagine a centralized system facing a standard newsvendor problem where a single decision maker operates the entire supply chain. Without loss of generality, we assume the uncertain demand X follows a continuous distribution with probability density function f, cumulative probability function F and complementary cumulative probability function $\overline{F} = 1-F(x)$ defined on support $[a,b]$ $(0 < a \le b)$. Let r be the unit inbound cost and w.l.o.g. $p=1$ be the normalized out-bound cost. Therefore, the imaginary decision-maker seeks to decide an inventory level x to maximize the profit $\Pi(x)$ of the entire supply chain

$$\max_{a \le x \le b} \Pi(x) \equiv -rx + \mathbb{E}[\min\{x, X\}] = \max_{a \le x \le b} \left(-rx + \int_0^x \overline{F}(t)\, dt \right)$$

with optimal order quantity x^c equal to the upper support b (when $\overline{F}(x) > r$ for all $x \in [a,b]$, namely the objective function increases within $[a,b]$) or uniquely determined by

$$\overline{F}(x^c) = r, \text{ for some } x^c \in [a,b].$$

2.2. The decentralized setting

The game-theoretic settings to be considered can be described by the following simple scenario with two players. There is a manufacturer producing a product with cost r per unit whose goal is to set the price of the product to some value w so that he maximizes his profit. There is also a retailer who buys the product from the manufacturer at price w per unit and sells it with price 1 per unit. The demand X for the product follows a probability distribution. Hence, the goal of the retailer is, given the price w of the product, to determine the inventory x that optimizes his expected profit, i.e., to determine x so that $-wx + \mathbb{E}[\min x, X]$ is maximized. Now, given the decision of the retailer for x, the manufacturer's optimal price is a value for w so that her profit $(w-r)x$ is maximized. When both the manufacturer and the retailer are profit maximizers, the total profit at equilibrium will be in general suboptimal. The PoA captures the profit loss due to the selfishness of the manufacturer and the retailer.

More complicated settings that generalize the one above will be considered in the sections to follow. They include those with more than two parties (Section 3) and with a tree-like structure (Sections 4 and 5).

2.3. The formal definition of PoA

Before we formally define PoA, we note an essential difference between the classes $\mathcal{F}[a,b]$ and $\mathcal{F}_{\text{IGFR}}$, which poses some technical challenges in our analysis later. The price-only contract can be formulated as a multi-level mathematical program, where multiple optimal solutions (equilibria) may exist in the lower level problem for some parameter values, leading to ambiguity in the definition of the problem. To avoid this ambiguity, this work adopts the well-accepted *optimistic* approach in the multi-level programming literature (e.g., [5]) with the economic interpretation that the follower is willing to support the leader, namely the follower will select, among all solutions optimal to himself, one that is best for the leader. Note that this is not a concern for the two-stage price-only contract under $\mathcal{F}[a,b]$ and two- or three-stage problem under $\mathcal{F}_{\text{IGFR}}$.

Throughout the rest of this paper, let x^c denote the optimal inventory level of the centralized system and x^d any inventory level of the decentralized system at equilibrium.

To capture the essence of the issues, we assume, where applicable, that $F(x)$ is smooth enough to ensure differentiability almost everywhere. For convenience we denote $\rho = b/a$ and

$$\alpha_F(x,y) := \int_x^y \overline{F}(t)\, dt, \quad \forall\ 0 \le x \le y,$$

dropping off the subscript F whenever no confusion is caused. Let us formally define the price of anarchy (PoA) for a given price-only contract as follows.

Definition 1.

$$\text{PoA} = \sup_{F \in \mathcal{F}[a,b]} \frac{\Pi(x^c)}{\min_{x^d} \Pi(x^d)} = \sup_{F \in \mathcal{F}[a,b]} \frac{-rx^c + \alpha_F(0,x^c)}{\min_{x^d}\{-rx^d + \alpha_F(0,x^d)\}}$$
$$= 1 + \sup_{F \in \mathcal{F}[a,b]} \min_{x^d} \frac{-r(x^c - x^d) + \alpha_F(x^d, x^c)}{-rx^d + \alpha_F(0,x^d)}.$$

3. Serial supply chain

The organization of this section is as follows: we first describe the problem in Section 3.1, then present the exact PoA in Section 3.2, and finally utilize the uniform distribution to show detailed different behaviors between the classes $\mathcal{F}[a,b]$ and $\mathcal{F}_{\text{IGFR}}$ in Section 3.3.

3.1. Problem description

Let us label the stages of the decentralized supply chain in an increasing order from downstream to upstream: $1, \ldots, n$. Each upstream stage i $(i = n, \ldots, 2)$ as a leader offers a wholesale price w_{i-1} to his next downstream stage $i-1$ as a follower, who accepts his offer as long as his expected profit is non-negative. The price-only contract under this supply chain system can be formulated as an n-level optimization problem (refer to Fig. 1):

Level 1. Stage 1 as the retailer with given transferring price w_1 offered by Stage 2, faces the random customer demand X and chooses his order quantity x as inventory in such a way that his profit is to be maximized after selling the products to customers at a unit price of $p=1$

$$\max_{a \le x \le b} \left(-w_1 x + \int_0^x \overline{F}(t)\, dt \right).$$

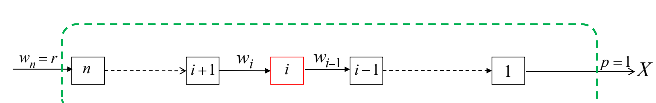

Fig. 1. Decentralized multistage supply chain with the upstream stages as leaders.