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# IV threshold cointegration tests and the Taylor rule

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ABSTRACT

The usual cointegration tests often entail nuisance parameters that hinder precise inference. This problem is even more pronounced in a nonlinear threshold framework when stationary covariates are included. In this paper, we propose new threshold cointegration tests based on instrumental variables estimation. The newly suggested IV threshold cointegration tests have standard distributions that do not depend on any stationary covariates. These desirable properties allow us to formally test for threshold cointegration in a nonlinear Taylor rule. We perform this analysis using real-time U.S. data for several sample periods from 1970 to 2005. In contrast to the linear model, we find strong evidence of cointegration in a nonlinear Taylor rule with threshold effects. Overall, we find that the Federal Reserve is far more policy active when inflation is high than when inflation is low. In addition, we reaffirm the notion that the response to counteract high inflation was weakest in the 1970s and strongest in the Greenspan era.



## 1. Introduction

A large and growing literature utilizes a threshold regression (TR) to capture the nonlinear relationships found among many macroeconomic variables. As in the linear regression framework, the estimation results from nonlinear regressions will be spurious if nonstationary *I*(1) variables are not cointegrated. In this regard, Balke and Fomby (1997) examined threshold cointegration by assuming that cointegration exists within a certain range of deviations from the long-run equilibrium implied by the null, but did not provide formal tests for threshold cointegration. Enders and Siklos (2001) provide critical values for threshold cointegration tests in a specific threshold specification that permits asymmetric adjustment in the error correction term. Nevertheless, testing for threshold cointegration is difficult when the distributions of the relevant test statistics depend on nuisance parameters. For example, the usual cointegration tests will depend on a nuisance parameter when stationary covariates are included, and the problem becomes even more pronounced in a nonlinear framework. However, bootstrapping the critical values does not appear a good solution in such cases. Enders et al. (2007) find that bootstrapping a test for persistence in a TR leads to excessively wide confidence intervals.

In this paper, we adopt a new methodology using instrumental variables (IV) estimation where, with one caveat, inference in a TR can be undertaken free of nuisance parameters. For this purpose, we extend the linear IV cointegration tests of Enders et al. (2009) and introduce new IV threshold cointegration tests can result in test statistics that can have standard normal, *t*, *F* or $\chi^2$ distributions. This outcome permits us to perform inference without the necessity of bootstrapping or using nonstandard distributions that depend on the particular model specification. In our methodology, the asymptotic distributions of threshold cointegration, weak-exogeneity, and symmetry tests are all standard even when stationary covariates are included. Monte Carlo experiments demonstrate that the IV threshold cointegration test has reasonable size and power properties.

Then, we apply our methodology to test for threshold cointegration in a nonlinear Taylor rule (Taylor, 1993). There are strong reasons to believe that modeling the Taylor rule is especially amenable to our methodology. Given a growing body of literature on testing nonlinear Taylor rules, it is somewhat surprising that no paper performs tests for nonlinear cointegration. However, this outcome may be due to difficulties found in the existing tests. To explain the issues involved, consider a standard linear Taylor rule specification:

$$\begin{aligned} i_t &= r^* + \pi_t + \alpha_1^*\left(\pi_t - \pi^*\right) + \alpha_2 y_t + \alpha_3 i_{t-1} + \alpha_4 i_{t-2} + \varepsilon_t \\ &= \alpha_0 + \alpha_1 \pi_t + \alpha_2 y_t + \alpha_3 i_{t-1} + \alpha_4 i_{t-2} + \varepsilon_t, \end{aligned} \tag{1}$$

where $i_t$ is the nominal federal funds interest rate, $r^*$ is the equilibrium real interest rate, $\pi_t$ is the average inflation rate over the previous four quarters, $\pi^*$ is the central bank's inflation target, $y_t$ is

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the "output gap" measured as the percentage deviation of real GDP from potential real GDP, $\alpha_0 = r^* - \alpha_1^* \pi^*$, $\alpha_1 = 1 + \alpha_1^*$, and $\varepsilon_t$ is an error term. The lagged terms $i_{t-1}$ and $i_{t-2}$ are included to allow for the possibility of interest rate smoothing, where adjustment to the target rate is gradual.

The recent macroeconometric literature suggests that simple OLS or GMM estimation of Eq. (1) may not be appropriate. For example, Bunzel and Enders (2010) and Österholm (2005) show that the federal funds rate and inflation rate act as unit root processes and the output gap is stationary. These papers employ a battery of Johansen (1988, 1991) cointegration tests and conclude that there is no meaningful linear cointegrating relationship between the inflation rate, output gap, and federal funds rate. We reconfirm similar results in this paper using real-time data. Concurrently, a growing body of literature suggests that the relationship between the federal funds rate, output gap, and inflation rate is likely to be some form of nonlinear regime-switching model; see, for example, the papers by Bec et al. (2002), Boivin (2006), Taylor and Davradakis (2006) and Qin and Enders (2008). Intuitively, we note that the model specification in Eq. (1) already shows possible instability in its underlying parameters. For example, the intercept term, $\alpha_0 = r^* - \alpha_1^* \pi^*$, can vary if the central bank's inflation target $\pi^*$ changes. Furthermore, to the extent that the Federal Reserve is more concerned about high inflation than low inflation, the response of $i_t$ is expected to be more dramatic when inflation is above the target rate than when inflation is below the target. Moreover, if it is more difficult for the Fed to reduce inflation than to increase inflation, the response of $i_t$ should be greater for positive values of $(\pi_t - \pi^*)$ than for negative values. Since similar arguments can be made regarding the relationship between $i_t$ and the output gap, it seems reasonable to modify Eq. (1) to estimate the relationship between $i_t$, $\pi_t$ and $y_t$ in a threshold framework. However, this poses an important fundamental question. In order to correctly estimate a Taylor rule with threshold effects, we must first know if a threshold cointegrating relationship exists.

Testing for threshold cointegration in a nonlinear Taylor rule is complicated by (a) the presence of the stationary covariate $y_t$, (b) the presence of the lagged interest rate terms, and (c) the possibility that the variables in the model are jointly endogenous. While a researcher might want to include $y_t$, $i_{t-1}$ and $i_{t-2}$ in a test for cointegration between $i_t$ and $\pi_t$ in order to reduce the estimated variance of the error term, including these variables will cause the test statistics to depend on nuisance parameters. Perhaps for these reasons, the literature has been silent in providing evidence of threshold cointegration in a nonlinear Taylor rule. Indeed, the extant literature does not contain a straightforward threshold cointegration test without nuisance parameters. As we will demonstrate, by including stationary IV in our tests we can conduct statistical inference concerning cointegration and threshold behavior in a nonlinear Taylor rule without the need to resort to a bootstrap procedure.

To preview our empirical findings, we show that the behavior of the Federal Reserve during the Burns–Miller period was very different from that during the Volcker and Greenspan periods. For each subsample beginning with the Paul Volcker era, our testing procedure indicates the presence of a significant threshold cointegrating relationship in a nonlinear Taylor rule. A particularly interesting result is that the Federal Reserve is far more policy active when inflation is high than when inflation is low. While these findings are robust to several different time periods, we find that the Federal Reserve was most aggressive to counteract inflation during the Greenspan era and least aggressive in the 1970s.

The paper proceeds as follows. In Section 2, we describe our testing methodology. The asymptotic properties are derived and finite sample properties are examined in simulations. Proofs are provided in Appendix A. In Section 3, we present our empirical findings of testing for threshold cointegration in a nonlinear Taylor rule. Concluding remarks are provided in Section 4.

## 2. Estimation and testing methodology

In this section, we present a general testing methodology for threshold cointegration that, with one caveat, avoids the nuisance parameter problem. Consider the following threshold autoregressive distributed lag (ADL) model:

$$\Delta x_{1t} = \left[\alpha_{11} x_{1,t-1} + \alpha_{12}' x_{2,t-1} + \phi_{11}' d_t + \phi_{12}' s_t\right] I_{1t} + \left[\beta_{11} x_{1,t-1} + \beta_{12}' x_{2,t-1} + \gamma_{11}' d_t + \gamma_{12}' s_t\right] I_{2t} + u_{1t} \tag{2a}$$

$$\Delta x_{2t} = \left[\alpha_{21} x_{1,t-1} + \alpha_{22}' x_{2,t-1} + \phi_{21}' d_t + \phi_{22}' s_t\right] I_{1t} + \left[\beta_{21} x_{1,t-1} + \beta_{22}' x_{2,t-1} + \gamma_{21}' d_t + \gamma_{22}' s_t\right] I_{2t} + u_{2t} \tag{2b}$$

$$I_{1t} = I(h_t > \tau) \text{and } I_{2t} = 1 - I_{1t}, \tag{2c}$$

where $x_t = (x_{1t}, \ldots, x_{pt})'$ is a $p$-dimensional $I(1)$ time series, $d_t$ includes a constant term (or all relevant deterministic terms) and lagged differenced terms that correct for serial correlation, $s_t$ includes one or more stationary right hand variables, and $u_t \sim N(0, \Sigma)$. $I_{1t}$ is a Heaviside indicator such that $I_{1t} = 1$ if $h_t > \tau$ and $I_{1t} = 0$ otherwise, where $h_t$ is the threshold variable (or function) and $\tau$ is the threshold value or parameter. If desired, we can allow for a delay parameter $d$, where $h_t$ is replaced throughout with $h_{t-d}$. Following Li and Lee (2010), we adopt percentiles of the threshold variable and consider different indicator functions where the threshold variable $h_t$ can be $I(0)$ as in Hansen and Seo (2002) or $I(1)$ as in Seo (2006). When $h_t$ is $I(0)$,

$$I_{1t} = I(h_t > \tau) = I(h_t > h_t^*(c)) \rightarrow I(U(r) > c), \tag{3a}$$

where $h_t^*(c)$ denotes the threshold value which is the $c$-th percentile of the empirical distribution of $h_t$ and $U(r)$ is a univariate process having a uniform distribution on $r \in [0,1]$. Alternatively, when $h_t$ is $I(1)$,

$$I_{1t} = I(h_t > \tau) = I\left(\sigma^{-1} T^{-1/2} h_t > \sigma^{-1} T^{-1/2} h_t^*(c)\right) \rightarrow I(W(r) > W^*(c)), \tag{3b}$$

where $W(r)$ is a Brownian motion on $r \in [0,1]$, $\sigma^2 = T^{-1} E(\sum \Delta h_t)^2$, $h_t^*(c)$ denotes the threshold value which is the $c$-th percentile of the empirical distribution of the normalized $h_t$, and $W^*(c)$ is a Brownian motion using a sorted time series evaluated at the $c$-th percentile. Thus, in both indicator functions, the threshold parameter is transformed into the percentile parameter. Note that unlike in Seo (2006), it is unnecessary to assume a fixed threshold value that vanishes asymptotically.[4]

In this paper, we will consider a single equation threshold version of the threshold ADL cointegration test. If the variable $x_{2t}$ in Eq. (2b) is weakly exogenous, it is necessary to add $\Delta x_{2t}$ as a regressor in the equation describing the conditional expectation of $\Delta x_{1t}$ given $\Delta x_{2t}$. Then, we can consider the following conditional model in a single equation testing regression:

$$\Delta x_{1t} = \left[\alpha_{11} x_{1,t-1} + \alpha_{12}' x_{2,t-1} + \phi_{11}' d_t + \phi_{12}' s_t + \phi_{13}' \Delta x_{2t}\right] I_{1t} + \left[\beta_{11} x_{1,t-1} + \beta_{12}' x_{2,t-1} + \gamma_{11}' d_t + \gamma_{12}' s_t + \gamma_{13}' \Delta x_{2t}\right] I_{2t} + u_{1t}. \tag{4}$$

[4] Note that Eqs. (2a) and (2b) differ from the usual threshold models, since they include one or more stationary right hand variables, $s_t$. In our analysis of the Taylor rule, we find that the output gap is stationary and we want to incorporate this information in our tests. Including stationary covariates in OLS-based cointegration tests is cumbersome, since the test statistics will critically depend on the nuisance parameter $\rho^2$ describing the long-run correlation between $u_t$ and $\nu_t$, where $\nu_t = \zeta' s_t + u_t$; see Zivot (2000) and Li (2006). This outcome is the same in nature as when adding stationary covariates to unit root tests, as initially suggested in Hansen (1995).