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Optimal allocation of trend following strategies

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HIGHLIGHTS

- Allocation of trend following strategies is studied within a Gaussian market model.
- Analytical formulas for the mean and variance of the portfolio return are derived.
- The optimal portfolio is constructed by introducing lead-lag corrections.
- Inter-asset correlations are not deteriorative but beneficial for trend followers.
- These concepts are illustrated on three highly correlated futures markets.

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ABSTRACT

We consider a portfolio allocation problem for trend following (TF) strategies on multiple correlated assets. Under simplifying assumptions of a Gaussian market and linear TF strategies, we derive analytical formulas for the mean and variance of the portfolio return. We construct then the optimal portfolio that maximizes risk-adjusted return by accounting for inter-asset correlations. The dynamic allocation problem for n assets is shown to be equivalent to the classical static allocation problem for n^2 virtual assets that include lead-lag corrections in positions of TF strategies. The respective roles of asset auto-correlations and inter-asset correlations are investigated in depth for the two-asset case and a sector model. In contrast to the principle of diversification suggesting to treat uncorrelated assets, we show that inter-asset correlations allow one to estimate apparent trends more reliably and to adjust the TF positions more efficiently. If properly accounted for, inter-asset correlations are not deteriorative but beneficial for portfolio management that can open new profit opportunities for trend followers. These concepts are illustrated using daily returns of three highly correlated futures markets: the E-mini S&P 500, Euro Stoxx 50 index, and the US 10-year T-note futures.

1. Introduction

For decades, market participants have attempted to detect potential trends in asset price fluctuations on exchange markets. In systematic trading, trend following (TF) strategies that generate buy or sell signals to adjust their market exposure according to past price variations, were developed to profit from trends at various time horizons [1–5]. While its actual profitability is highly controversial [6–9], trend following remains a widely used strategy among professional asset managers. Since many traders search for the same profit opportunities, the expected (net) gains are small, especially at short

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times, and are subject to stochastic fluctuations. In order to enhance profit and reduce risk, fund managers build diversified portfolios, aiming to decorrelate constituent TF strategies as much as possible. Our goal is to show that this conventional approach leads to sub-optimal portfolios. In particular, we illustrate that inter-asset correlations, if accounted for properly, facilitate trend detection and thus significantly improve the risk-adjusted portfolio return.

In a previous work, we considered a linear TF strategy applied to an asset with auto-correlated returns [10]. In order to study trend following from a risk–reward perspective, we introduced an explicit persistence in asset returns by adding a stochastic trend term to a Gaussian market model. This simple framework allowed us to derive analytical formulas for the mean and variance of the strategy profit-and-losses (P&L). We were able to derive a threshold in auto-correlation below which a trend follower has no hope to realize profit given market transaction costs. Fund managers often use such criteria to select a set of assets/markets that are of interest for trend following trading. Many examples of TF strategies applied to stock markets, foreign exchange markets, and commodities were reported [2,11–13]. In the financial industry, diversified funds apply TF strategies to a large number of assets with hope to benefit from the so-called diversification effect [14,15].

In this paper we extend the model from Ref. [10] to the multivariate case. We introduce an asset covariance structure into a stochastic trend process and study how correlations between stochastic trends affect the portfolio risk–reward profile. We solve the portfolio optimization problem by taking into account the trend following nature of trading strategies. Our goal is to show that failure to account for trend correlations (i.e., only using asset returns covariance) leads to sub-optimal risk-adjusted portfolio return.

Modern portfolio theory [16–20], initiated by Markowitz [21], offers numerous solutions to the asset allocation problem [22]. The original problem was to find portfolio weights, i.e. amount of capital allocated to each asset, that maximize a portfolio mean–variance objective given expected market returns and covariance structure. In our approach, we consider a similar problem for TF strategies. The expected return of TF strategy depends directly on the asset expected excess variance (or auto-correlation) that characterizes market trends [10,23]. We solve the problem of static allocation of dynamic strategies by specifying the correlation structure of trend and noise components of asset price fluctuations.

We keep simple modeling assumptions from Ref. [10] in an effort to derive an exact form of the optimal allocation. We model price variations as a stochastic trend added to a white noise and we introduce separately the correlation in trends and the correlation in noises. For instance, two assets can exhibit similar long-term trends and be negatively correlated on the short term. Under these assumptions, we find that the static allocation problem in which an optimal weight is assigned to each asset, leads to sub-optimal risk-adjusted portfolio return. Even if the correlations in trends and noises are equal, the application of a classical Markowitz approach to TF strategies is sub-optimal. We then formulate a dynamic allocation framework that leads to an improved risk-adjusted return of the portfolio. Our approach to dynamic allocation consists in correcting each strategy signal by a linear combination of other strategy signals. This cross-correcting term can be seen as a lead–lag correction [24,25]. We show that the allocation problem for n dynamic strategies can be reduced to solving a static Markowitz problem for a set of n^2 virtual assets with explicitly derived expected returns and covariance structure.

The paper is organized as follows. In Section 2, we introduce the standard mathematical tools to solve the portfolio allocation problem. In Section 3, we study in detail the two-asset portfolio problem while Section 4 extends to the case of multiple assets with identical correlations (e.g., a sector of the market). We quantify the improvement in terms of the expected Sharpe ratio (or risk-adjusted return) of the portfolio and the Sharpe gain compared to a static allocation scheme. In Section 5, we discuss some challenges in developing inference tools for estimating the model parameters from empirical data and illustrate the effect of the lead–lag correction onto the profitability of TF portfolios for futures markets. Conclusion section summarizes the main results, while technical derivations are reported in Appendices.

2. Market model and trading strategy

We first introduce a mathematical market model for n assets and describe linear trend following strategies. We then present the dynamic portfolio allocation based on a linear combination of strategy signals. In this frame, we derive mean and variance of portfolio returns, formulate the optimal allocation problem, and show its reduction to a standard static allocation problem for n^2 virtual assets.

2.1. Market model

We assume that the return[1] r_t^j of the jth asset at time t has two contributions: an instantaneous fluctuation (noise) ε_t^j, and a stochastic trend which in general is given as a linear combination of random fluctuations $\xi_{t'}^j$,

$$r_t^j = \varepsilon_t^j + \sum_{t'=1}^{t-1} \mathbf{A}_{t,t'}^j \xi_{t'}^j, \tag{1}$$

[1] Throughout this paper, daily price variations are called "returns" for the sake of simplicity. Rigorously speaking, we consider additive logarithmic returns resized by realized volatility which is a common practice on futures markets [26,27]. Although asset returns are known to exhibit various non-Gaussian features (so-called "stylized facts" [28–33]), resizing by realized volatility allows one to reduce, to some extent, the impact of changes in volatility and its correlations [34,35], and to get closer to the Gaussian hypothesis of returns [36].