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A mixed-integer LP model for the optimal allocation of voltage regulators and capacitors in radial distribution systems

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ABSTRACT

This paper presents a mixed-integer linear programming model to solve the problem of allocating voltage regulators and fixed or switched capacitors (VRCs) in radial distribution systems. The use of a mixed-integer linear model guarantees convergence to optimality using existing optimization software. In the proposed model, the steady-state operation of the radial distribution system is modeled through linear expressions. The results of one test system and one real distribution system are presented in order to show the accuracy as well as the efficiency of the proposed solution technique. An heuristic to obtain the Pareto front for the multiobjective VRCs allocation problem is also presented.

1. Introduction

The voltage regulation is an important function of an electrical distribution system (EDS) and is the ability of to provide a voltage magnitude within standard ranges for a wide range of load conditions [1]. The voltage magnitude ranges are imposed by the electricity regulatory agency of each country to guarantee a quality service delivery to consumers. Thus the utilities are obliged to maintain an adequate voltage profile in EDS, and this require investments in appropriate devices that must be economically viable. The most common way to regulate the voltage magnitude in EDS is to install voltage regulators and capacitors, evaluating several aspects like installation cost, equipment utilization rate, quality of service and loss minimization [2].

The installation of capacitors in an EDS is important for providing reactive power support, power factor correction, voltage profile improvement and loss minimization [3]. Therefore, the location, the size and the number of the equipments to be installed in an EDS must be identified in order to ensure quality service. Much research about the capacitor allocation (CA) problem can be found in the literature, as shown in [4]. The CA problem is commonly modeled as a mixed integer nonlinear programming (MINLP) problem [5–8], where the objective function is usually to minimize investment costs plus the power losses costs in EDS [6,9]. Among the methodologies used to solve the CA problem can be found constructive heuristic algorithms [5,8,10]; metaheuristics like the genetic algorithms [11,12], tabu search [13], plant growth simulation [14] and particle swarm [15]; and classical approaches were used to solve the CA problem like the branch and bound algorithm [16], where the CA problem is modeled like a mixed integer linear programming (MILP) problem using current injection equations considering the constant current type load.

The installation of voltage regulators in an EDS is also important for controlling the voltage profile mainly in large feeders and/or with large loads at the feeder's end, where the greatest problems of voltage drop occur [17]. Just as in the CA problem, the voltage regulators allocation (VRA) problem can also be modeled as a MINLP problem, where the objective function is usually to minimize the investment costs plus the active losses costs in an EDS [17,18]. In [19] the voltage magnitude deviation is also considered in the objective function. Among the methodologies used to solve the VRA problem it can be found constructive heuristic algorithms [17,18] and metaheuristics like the genetic algorithms [19] where a sweep load flow is used to calculate the steady-state operation point.

In most cases, the VRA problem is solved separately from the CA problem. For example, in [20–22] the problem of location, sizing and control of capacitors and the problem of location and control of voltage regulators are considered but are solved separately in two decoupled problems. Only in [23–27] methodologies are presented to jointly solve the problem of allocating voltage regulators and capacitors by using genetic algorithms [23–26] and tabu search [27]. In [23–25] the problem of allocation, sizing and type (fixed or switched) of capacitors and allocation and configuration

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Nomenclature

Sets

Ω_b	sets of nodes
Ω_l	sets of branches
Ω_d	sets of load levels
Ω_r	sets of voltage regulator types

Constants

κ_c	capital recovery rate of capacitor constructions
κ_r	capital recovery rate of voltage regulator constructions
c_d^{ls}	energy cost in load level d (US$/kW h)
c_r^{vr}	annualized installation cost of the voltage regulator type r (US$)
c^{un}	annualized unit cost of each standard capacitor unit (US$)
c^{sw}	annualized switch equipment cost of the switched capacitors (US$)
c^{fx}	annualized installation cost of the fixed and switched capacitors (US$)
α_d	number of hours of load level d in one year (h)
τ_l	interest rate of the cost of power losses
$\bar{n}^{cp}$	maximum number of fixed and switched capacitors that can be added in the system
$\bar{n}_b^{cp}$	maximum number of standard capacitor units that can be installed in a node of the system
$\bar{n}^{vr}$	maximum number of voltage regulators that can be added in the system
$\underline{V}$	minimum voltage magnitude (kV)
$\overline{V}$	maximum voltage magnitude (kV)
$\bar{I}v_r$	maximum current magnitude of voltage regulator type r (A)
$\bar{I}_{ij}$	maximum current magnitude of branch ij (A)
$P_{i,d}^D$	real power demand at node i in load level d (kW)
$Q_{i,d}^D$	reactive power demand at node i in load level d (kVAr)
R_{ij}	resistance of branch ij (Ω)
X_{ij}	reactance of branch ij (Ω)
Z_{ij}	impedance of branch ij (Ω)
R	number of blocks of the piecewise linearization
S	number of discretizations of the variable $\widetilde{V}_{i,d}^{sqr}$
$2\overline{nt}$	maximum step number of the voltage regulator tap
$R\%$	regulator range of voltage regulators
Q^{cp}	reactive power of each standard capacitor unit (kVAr)
$\overline{\Delta}^V$	discretization step of $\widetilde{V}_{i,d}^{sqr}$
$m_{ij,r}^S$	slope of the rth block of the power flow of branch ij
$\overline{\Delta}_{ij}^S$	upper bound of each block of the power flow of branch ij

Variables

q_i	binary variable for allocation of a fixed or switched capacitor at node i
q_i^{sw}	binary variable for allocation of a switch equipment for the capacitor units at node i
$v_{ij,r}$	binary variable for allocation of a voltage regulator on branch ij of type r
$nt_{ij,d}$	integer step number of the tap of the voltage regulator on branch ij in load level d
$n_{i,d}$	integer number of standard capacitor units operating at node i in load level d
n_i^{cp}	number of standard capacitor units installed at node i
$t_{ij,d}$	voltage regulator tap of branch ij in load level d
$t_{ij,d}^{sqr}$	square of $t_{ij,d}$
$P_{ij,d}$	real power flow of branch ij in load level d
$Q_{ij,d}$	reactive power flow of branch ij in load level d
$P_{i,d}^S$	real power provided by substation at node i in load level d
$Q_{i,d}^S$	reactive power provided by substation at node i in load level d
$V_{i,d}$	voltage magnitude at node i in load level d
$V_{i,d}^{sqr}$	square of $V_{i,d}$
$\widetilde{V}_{i,d}$	non-regulated voltage magnitude at node i in load level d
$\widetilde{V}_{i,d}^{sqr}$	square of $\widetilde{V}_{i,d}$
$I_{ij,d}$	current flow magnitude of branch ij in load level d
$I_{ij,d}^{sqr}$	square of $I_{ij,d}$
$x_{j,d,s}$	binary variable used in the discretization of $\widetilde{V}_{i,d}^{sqr}$
$P_{j,d,s}^c$	power corrections used in the discretization of $\widetilde{V}_{i,d}^{sqr} I_{ij,d}^{sqr}$
$V_{j,d,s}^c$	voltage magnitude corrections used in the discretization of $t_{ij,d}^{sqr} \widetilde{V}_{i,d}^{sqr}$
$\Delta_{ij,d,r}^P$	value of the rth block of $\lvert P_{ij,d}\rvert$
$\Delta_{ij,d,r}^Q$	value of the rth block of $\lvert Q_{ij,d}\rvert$

of voltage regulators, considering different load conditions, are modeled as a MINLP problem. In [26] the problem of allocation and coordination of voltage regulators and capacitors considers the impact on the harmonic distortion of bus voltage in three-phase electrical distribution system. The problem of allocation and coordination of voltage regulators and capacitors for the voltage profile control considering the presence of distributed generation in an EDS is presented in [27].

Although the capacitors can contribute to improve the voltage profile, an EDS can reach a state in which full reactive support exists and problems of voltage drop still occur [20]. In this cases, the allocation of voltage regulators, which can provide a better control of the voltage regulation and reduce the operation cost, becomes necessary. Therefore, the joint allocation of voltage regulators and capacitors has the advantage of assessing which set of equipments is the most appropriate to be installed in an EDS.

In this paper, the voltage regulators and fixed or switched capacitors (VRCs) allocation problem in radial distribution systems is modeled as a mixed integer linear programming problem. The proposed model allows the independent or joint solution of the VRCs allocation problems. Linearizations were made to adequately represent the steady-state operation of the EDS considering the behavior of the constant power type load. The integer nature of the decision variables represents the allocation, size and type of voltage regulators and capacitors. The objective is to minimize the total investment and operation costs subject to operation and physical constraints. The proposed model was tested in systems of 69 and 136 nodes. In order to validate the linearizations performed, the steady-state operation point was also calculated using a load flow sweep method. The main contributions of this paper are as follows:

1. A novel model for the steady-state operation of a radial distribution system through the use of linear expressions.
2. A mixed integer linear programming model for the VRCs allocation problem that presents an efficient computational behavior with conventional MILP solvers.
3. A heuristic to obtain the Pareto front for the VRCs allocation problem considering two different objective functions (total cost and maximum voltage deviation).