Electric Power Systems Research 80 (2010) 642–650

Contents lists available at ScienceDirect

Electric Power Systems Research

journal homepage: www.elsevier.com/locate/epsr

Optimal allocation of capacitors in unbalanced multi-converter distribution systems: A comparison of some fast techniques based on genetic algorithms

G. Carpinelli[a], D. Proto[a,*], C. Noce[b], A. Russo[c], P. Varilone[d]

[a] Università degli Studi di Napoli Federico II, Via Claudio, 21, 80125 Naples, Italy
[b] ENEL Distribuzione, Rome, Italy
[c] Politecnico di Torino, Turin, Italy
[d] Università degli Studi di Cassino, Cassino, Italy

ARTICLE INFO

Article history:
Received 16 July 2008
Received in revised form 30 April 2009
Accepted 25 October 2009
Available online 3 December 2009

Keywords:
Optimal allocation of capacitors
Genetic algorithms
Inherent Structure Theory of Networks
Sensitivity analysis
Micro-genetic algorithms

ABSTRACT

Genetic algorithms (GAs) are widely used for optimal allocation of capacitors in distribution systems. When dealing with large-scale systems, such as in case of unbalanced multi-converter distribution systems, these algorithms can require significant computational efforts, which reduce their effectiveness. In order to reduce processing time for GAs and simultaneously maintain adequate levels of accuracy, methods based on the reduction of the search space of GAs or based on micro-genetic algorithms have been proposed. These methods generally guarantee good solutions with acceptable levels of computational effort. In this paper, some fast, GA-based methods are compared and applied for solving the problem of optimal sizing and siting of capacitors in unbalanced multi-converter distribution systems. The algorithms have been implemented and tested on the unbalanced IEEE 34-bus test distribution system, and their performances have been compared with the performance of the simple genetic algorithm technique.

1. Introduction

In this paper, the problem of determining the optimal sizing and location of capacitors in distribution systems is considered. This is a typical example of a large-scale, mixed, non-linear, constrained, optimization problem, since the solution should take into account various operational and equipment limits.

The objective of the capacitor placement is either to minimize the total costs (i.e., costs of capacitors and losses) or to optimize the voltage profile, and to make sure that these capacitors will have the minimum impact on the waveform distortion of the bus voltages in the distribution system. This is due to the possibility of unwanted over-voltages that may be caused by harmonic resonance due to the presence of static converters in the system. Furthermore, the facts that distribution systems can operate with unbalanced loading conditions and can be characterized by the presence of feeders with missing phases mean that the optimization will have to account for any unbalances in the system.

Then, the optimal placement of capacitors should be formulated with reference to an unbalanced multi-converter distribution system.

Such problems can be solved by using GAs that have been proven to find good solutions for sizing and locating capacitors [1–9]. However, as the system size increases, as is the case for unbalanced systems in which the waveform distortions have to be taken into account, the solution of this problem by GAs requires extensive calculation capacity and time, making efforts to produce fast solution procedures important.

In order to reduce processing time while maintaining reasonable accuracy, methods based on the reduction of GA search space or micro-genetic algorithms can be used.

A reduced feasible region, i.e., the search space, can be determined by means of sensitivity structures [8,9] or by fuzzy logic [10]; therefore, the optimization problem is faced in two steps, i.e., (1) determine the reduced feasible region (i.e., the reduced set of candidate busbars for the capacitor location) and (2) find the optimal solution in this reduced region, f.i. applying a simple genetic algorithm.

In this paper, two different approaches for the reduction of the feasible region are assessed. The first approach is based on the Inherent Structure Theory of Networks (ISTN); this Theory, which is based on the spectral representation of the admittance matrix, helps identify the individual candidate busbars at which the connection of a capacitor will give the maximum improvement of the voltage profile. The ISTN is based on the reformulation of network admittance and impedance matrices with respect to their eigenvalues and eigenvectors, and the Theory was first proposed by

* Corresponding author. Tel.: +39 0817683212; fax: +39 0812396897.
E-mail address: danproto@unina.it (D. Proto).

0378-7796/$ – see front matter © 2009 Elsevier B.V. All rights reserved.
doi:10.1016/j.epsr.2009.10.029

Laughton in [11,12]; also, recently, the Theory has been extended to solve other important problems in the field of power systems [13–16].

The second approach is based on the sensitivity of the power losses to the nodal injection of reactive power; the candidate busbars are the ones exhibiting the maximum sensitivity factors [8,9].

In addition to the aforementioned techniques that are based on reduction of search space, in this paper, we also apply micro-genetic algorithms (μGAs) to solve the problem of the optimal capacitor allocation. It has been proven that micro-genetic algorithms quickly arrive at the near optimal solution using a small population [6].

The ISTN-based approach, the sensitivity-based approach, and the micro-genetic algorithms considered in this paper have been applied in the past for the placement of capacitors in distribution systems without considering, as far as we know, waveform distortions and unbalances or, at least, considering either waveform distortions or unbalances [8–10,15].

Then, the original contributions of this paper are that it:

(i) extends the above-mentioned techniques to unbalanced multi-converter distribution systems in order to take into account contemporaneously both waveform distortions and unbalances; and
(ii) compares the behavior of the techniques with each other and with a simple GA (SGA) without any reduction of the search space, in order to outline their specificity and effectiveness.

Preliminary results on the above formulations have been presented in [17]. In this paper, we extend the details of the approaches shown in [17] and analyze more case studies with the purpose of comparing the GA-based methods for capacitor location.

The paper is organized as follows. The mathematical formulation of the optimization problem for the sizing and siting of capacitors is recalled with reference to the unbalanced distribution systems with linear and non-linear loads. Then, the techniques applied to obtain good solutions with tolerable computational efforts are described. Finally, numerical applications are provided to demonstrate the usefulness of the considered procedures. In addition, a comparison of the placement of capacitors obtained by studying the unbalanced system with a single-phase representation is also provided.

2. Optimal sizing and siting of shunt capacitors

Let us consider an unbalanced multi-converter distribution system. The problem of the allocation of capacitors can be formulated as a mixed non-linear constrained optimization problem in which an objective function has to be minimized while meeting a number of equality constraints (such as three-phase power flow equations) and inequality constraints (such as the admissible range of the bus phase voltages at the fundamental frequency, maximum values of individual voltage harmonics, and line currents at the fundamental frequency).

In particular, in this paper, the problem of the siting and sizing of capacitors in unbalanced distribution systems with linear and non-linear loads is formulated as:

$$\min F(\boldsymbol{X}, \boldsymbol{U}) \tag{1}$$

subject to:

$$\boldsymbol{g}(\boldsymbol{X}, \boldsymbol{U}) = 0 \tag{2}$$

$$\boldsymbol{h}(\boldsymbol{X}, \boldsymbol{U}) \leq 0, \tag{3}$$

where $\boldsymbol{X}$ is the system state vector (magnitudes and arguments of the phase voltages), and $\boldsymbol{U}$ is the capacitor units vector, placed at each bus.

In the following sections, more details about the objective function F, the equality constraints $\boldsymbol{g}$, and the inequality constraints $\boldsymbol{h}$ are given.

2.1. The objective function

In the most general case, when dealing with the problem of capacitor siting and sizing in distribution systems, several objectives should be met. In this paper, two objective functions are considered; the first, referred to as F_1, accounts for the total costs as:

$$F_1 = F_C + F_L, \tag{4}$$

where F_C is the cost of capacitors, and F_L is the cost of the losses evaluated at the fundamental and harmonic frequencies. More details on this topic can be found in [18].

The other objective function, referred to as F_2, accounts for the profile of voltage at all busbars, as shown in Eq. (5) [19]:

$$F_2 = \frac{\sum_{i=1}^{N_{bus}} \sum_{p=1}^{N_{ph,i}} (V_i^p - V_{nom})^2}{\sum_{i=1}^{N_{bus}} N_{ph,i}}, \tag{5}$$

where V_{nom} is the rated phase voltage, V_i^p is the amplitude of voltage at busbar i with phase p, $N_{ph,i}$ is the number of phases at busbar i, and N_{bus} is the number of considered busbars.

2.2. The equality constraints

Each solution of the minimization problem should satisfy the equality constraints, i.e., the three-phase power balance equations at the fundamental and harmonic frequencies.

At the fundamental frequency, the equality constraints are the well-known, three-phase, load-flow equations [20]. Phase coordinates are used so that voltages, powers, and currents are represented by a three-element vector (one for each phase); also, a three-phase representation of all system components is used so that generators, transformers (with all the possible winding connections), and lines are represented by 3 × 3 admittance submatrices.

At harmonic frequencies, once again, phase coordinates are used, and the linear harmonic equations for the three-phase network are included in the equality constraints:

$$\dot{\boldsymbol{Y}}_n \bar{\boldsymbol{V}}_n = \bar{\boldsymbol{I}}_n, \tag{6}$$

where $\dot{\boldsymbol{Y}}_n$, $\bar{\boldsymbol{V}}_n$, and $\bar{\boldsymbol{I}}_n$ are the three-phase network admittance matrix, the bus voltage vector, and the independent current source vector, respectively, evaluated at the nth harmonic order. The harmonic injections of non-linear devices are modeled according to the recommendations of the IEEE PES Working Group on Harmonics Modeling and Simulation [21,22], which suggested that the phase angles of the injected harmonic current sources should be adjusted according to the phase angle of the fundamental with respect to the reference. The same IEEE PES Working Group suggested component modeling used for harmonic representation [21,22].

2.3. The inequality constraints

Each solution should satisfy the considered limits of bus voltages and line currents at the fundamental frequency and of unbalance and harmonic distortion indices (unbalance factors, single voltage harmonics, and total harmonic distortions).

In addition, the constraints on the discrete nature of capacitor sizes are considered for both fixed (always connected) and switched (not always connected) types. Also, limits on the maximum number of capacitors allowed at each bus can be included.