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Single-objective probabilistic optimal allocation of capacitors in unbalanced distribution systems

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ABSTRACT

The optimal allocation of capacitors in unbalanced distribution systems can be formulated as a mixed integer, non-linear, constrained optimisation problem. Fuzzy-based approaches, simulated annealing, tabu search and genetic algorithms are some of the techniques used for solving the problem in deterministic scenarios. However, distribution systems are probabilistic in nature, leading to inaccurate deterministic solutions. As a result, a probabilistic optimization model is required to take into account the unavoidable uncertainties affecting the problem input data, primarily the load demands.

Of the various techniques for the solution of the problem, one of the most frequently used is the genetic algorithm. However, the application of simple genetic algorithms to solve the probabilistic optimization model involves tremendous computational effort. To reduce the computational effort, this paper proposes a new single-objective probabilistic approach based on the use of a micro-genetic algorithm. Two different techniques, one based on the linearised form of the equality constraints of the probabilistic optimisation model and one based on the point estimate method, were tested and compared. The proposed approaches were tested on the IEEE 34-node unbalanced distribution system to demonstrate the effectiveness of the procedures in generating reduced computational efforts and increased accuracy of the results.

1. Introduction

Shunt capacitors are commonly used in distribution systems to reduce power losses, improve the voltage profile along the feeders and increase the maximum flow through lines and transformers. Deciding on the optimal sizes and locations of these capacitors presents a complex problem; the sizes and locations are typically determined by solving optimisation problems in which proper objective functions are minimised while equality and inequality constraints are met. Both single-objective and multi-objective optimisation approaches have been proposed in the relevant literature. Single-objective approaches attempt to minimise only one objective function (either technical, as in the case of the voltage profile, or economical, as in the case of the total cost). Multi-objective approaches attempt to simultaneously minimise several objective functions describing both economical and technical issues; they can be used to find a set of feasible solutions (Pareto optimal solutions), among which the Decision Maker (DM) makes a choice. When an optimisation problem has a single objective, the definition of "best solution" is very simple, immediate and one-dimensional because there is only a single best solution (or none, eventually), and no DM choices are required. In contrast, a multi-objective problem with conflicting objectives has no single solution but instead has a set of optimal solutions. In this case, the multidimensional concept of "dominance" is used to determine whether one solution is better than the others, and the DM is allowed some discretion in setting parameters and choosing the final solution [1].

Both approaches are sharable and have advantages. However, in this paper, the single-objective approach is applied because it is often more practical from the Distribution System Operator (DSO) point of view, which can directly and easily identify which locations and sizes are beneficial for system operation.

The single-objective optimisation problem is widely recognised in the relevant literature for both balanced and unbalanced distribution systems, and several algorithms have been proposed to solve it. One of the algorithms most frequently used is the genetic algorithm (GA); it has been shown to be capable of finding good solutions, although it can require high computational effort [2–9].

In the GA application for solving capacitor placement optimisation problems, the load power variations with time are approximated by two or more (rarely more than three) discrete constant levels corresponding, in the case of a two-level curve, to

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the peak and off-peak levels. The time period to be studied, T, is consequently divided into intervals (for example T_1 and T_2) during which the load level is assumed to be *constant* versus time. The solution of the optimisation problem consists of determining the locations and sizes of the capacitors to be installed for each level; these capacitors are installed during the time interval corresponding to the duration of each level (T_1 and T_2). This is possible because fixed and switchable capacitors can be installed on busbars of real-world electrical distribution systems. Fixed capacitors are *always connected* at the same bus whatever the load level might be, while switched capacitors are *not always connected* but can be connected or disconnected *from the same bus* by the utility on the basis of the involved load level [10].

Because load time variations occur, however, the best approach to the solution of the capacitor allocation problem should consider the input variables (mainly the load demands) as random variables. The problem to be solved, therefore, becomes probabilistic, and the objective function as well as the equality and inequality constraints involves random variables; thus, a probabilistic approach to the problem solution is mandatory. In particular, in each level (for example, the peak and off-peak levels previously described), the load power should not be considered constant but instead should be characterised by a probabilistic function to account for the unavoidable time-varying nature of the load demands during these intervals. In practice, *for each level* (for example the peak or the off-peak level) the load demands are characterised either by joint probability density functions (the active and reactive powers are correlated random variables) or by marginal probability density functions (the active and reactive powers are uncorrelated random variables).

Unfortunately, while a myriad of papers proposing solutions to deterministic scenarios have been published (see, for example Refs. [1–9]), and, to the best of our knowledge, the vendors of distribution system analysis software have provided one or more well-consolidated algorithms that assume constant load levels, few papers in the technical literature have dealt with probabilistic approaches to optimal allocation in distribution systems with GAs.

In [11], a GA was used to optimise a stochastic voltage security index. In [12], a simple GA was applied to a reduced feasible region obtained through the Inherent Structure Theory of Networks. In both cases, the primary research effort was devoted to reducing the computational effort required by the application of GAs in solving the probabilistic optimisation model. A further contribution was provided in [13], a study in which the optimal reactive power control was obtained for a network with wind farms; in particular, a method based on cumulantes and Gram–Charlier series expansion was applied to calculate the probabilistic bus voltage. The Gray code was then used for GAs. Finally, in [14] a probabilistic approach to the optimal allocation of capacitors was proposed in the frame of a multi-objective optimisation. Due to the single-objective formulation, this paper differs substantially from [14] in both the probabilistic approach used for the optimisation model and the solution procedure; moreover, in [14], only the linearisation of the constraints was used to reduce computational efforts, whereas in this paper, in addition to linearization, the point estimate method [15] is tested.

Overall, the primary contribution of this paper consists of the proposal of a new probabilistic approach to the single-objective optimal allocation of capacitors in a distribution system. The approach is based on a micro-genetic algorithm (μGA) that operates on a small-size population, thus arriving quickly at a near-optimal solution. The μGA solves a probabilistic optimisation model that takes into account the uncertainties of the input random variables. To further reduce the computational effort, two techniques for calculating the statistical features of the state and dependent random variables were compared. The first technique is based on the linearisation of the equality constraints, while the second applies the point estimate method [15–19] as an alternative to the classical Monte Carlo simulation approach. To our knowledge, the point estimate method has not previously been used to solve a single-objective probabilistic optimal capacitor placement problem. The techniques were compared in terms of the accuracy of the results and the computational effort to ascertain which approach was most useful for the problem in the study. Finally, because electrical distribution systems usually contain a mixture of single-, double- and three-phase lines and loads, the work reported in this paper takes unbalanced distribution systems into consideration [20].

The relevance of the new probabilistic approach proposed in this paper is consistent with recent trends in the relevant literature in which much attention has been devoted to probabilistic approaches in solving planning and operation problems in power system analysis [21–23].

The remainder of this paper is organised as follows. Section 2 presents the probabilistic formulation of the optimal allocation of capacitors in an unbalanced distribution network, while Section 3 details the solution procedure. An application of the IEEE 34-node unbalanced test system is presented in Section 4, in which the results of the application are discussed and compared with those obtained using the deterministic approach.

2. Problem description

The deterministic problem of the location and sizes of capacitors is a mixed integer, non-linear, constrained optimisation problem in which an objective function (e.g., the voltage profile or the total cost including the losses cost) has to be minimised while a number of equality constraints (e.g., power flow equations) and inequality constraints (e.g., admissible ranges of the bus voltages and limits on the unbalanced factors and line currents) are met. The problem is particularly complex due to the discrete nature of both the locations (busbars) and sizes (multiple of a capacitor unit) of capacitors.

In similar models in the literature, a number of deterministic load levels are taken as references in the formulation of the optimisation problem. The time period of the study is divided into intervals of assigned duration, each of which is characterised by a *constant* load level. An optimal set of capacitors is calculated for each time interval to obtain the best performance by placement of capacitors over the whole planning period. This is possible because fixed and switchable capacitors are installed on busbars of real-world electrical distribution systems.

Because of the time-varying nature of the load demands inside each time interval, however, the best approach to the solution of the capacitor allocation problem should consider the input variables (mainly phase-load demands) to be random variables. Then, a probabilistic formulation of the problem should be adopted to take into account the random nature of the power required by the loads. In particular, in this paper, *for each load level*, the load powers are not assumed to be *constant*; instead, their random variations versus time are characterised by normal distributions. In particular, *for each level* (for example the peak or the off-peak level), the load demands are characterised either by joint probability density functions (active and reactive powers are correlated random variables) or by marginal probability density functions (active and reactive powers are uncorrelated random variables).

It should be noted that, although in probabilistic studies the load characteristics can be assumed to have either a normal distribution or a binomial, discrete, or other distribution, modelling based on a normal distribution is the most frequently used. For example, with reference to probabilistic balanced power flows, [23] reports an