ELSEVIER

Reliability Engineering and System Safety 76 (2002) 245–254

RELIABILITY ENGINEERING & SYSTEM SAFETY

www.elsevier.com/locate/ress

Optimal allocation of elements in a linear multi-state sliding window system

Gregory Levitin*

Reliability Department, Planning, Development and Technology Division, The Israel Electric Corporation Ltd, P.O. Box 10, 31000 Haifa, Israel

Received 19 September 2001; accepted 17 January 2002

Abstract

This paper proposes a new model that generalizes the consecutive *k*-out-of-*r*-from-n:F system to multi-state case. In this model (named linear multi-state sliding window system) the system consists of *n* linearly ordered multi-state elements. Each element can have different states: from complete failure up to perfect functioning. A performance rate is associated with each state. The system fails if the sum of the performance rates of any *r* consecutive elements is lower than a demand *W*.

An algorithm is suggested that finds the order of elements with different characteristics within linear multi-state sliding window system, which provides the greatest possible system reliability. The algorithm is based on using a universal generating function technique for system reliability evaluation. A genetic algorithm is used as the optimization tool. Illustrative examples are presented.

Keywords: Sliding window system; Consecutive *k*-out-of-*r*-from-n:F system; Multi-state element; Universal moment generating function; Genetic algorithm

1. Introduction

The linear consecutive *k*-out-of-*r*-from-n:F system has *n* ordered elements and fails if at least *k* out of *r* consecutive elements fail. This system was formally introduced by Griffith [1], but had been mentioned previously by Tong [2], Saperstain [3,4], Naus [5] and Nelson [6] in connection with tests for non-random clustering, quality control and inspection procedures, service systems, and radar problems. Different algorithms for evaluating the system reliability were suggested in Refs. [7–10].

When $r = n$, one has the well studied simple *k*-out-n:F system. When $k = r$, one has the consecutive *k*-out-of-n:F system, which was introduced by Chiang and Niu [11], and Bollinger [12,13]. The simple *k*-out-n:F system was generalized to the multi-state case by Wu and Chen in Ref. [14], where system elements have two states but can have different integer values of nominal performance rate. In Ref. [15], the general model is developed in which elements can have an arbitrary number of real-valued performance levels. The multi-state generalization of the consecutive *k*-out-of-n:F system was first suggested by Hwang and Yao [16] as a generalization of linear consecutive-*k*-out-of-n:F system and linear consecutively connected system with 2-state elements, studied by Shanthikumar [17,18].

This paper considers a new model that generalizes the consecutive *k*-out-of-*r*-from-n:F system to the multi-state case. In this model (named linear multi-state sliding window system) the system consists of *n* linearly ordered multi-state elements (MEs). Each ME *j* can have H_j different states: from complete failure up to perfect functioning. A performance rate is associated with each state. The SWS fails if the sum of the performance rates of any *r* consecutive MEs is lower than the demand *W*.

Note that the special case of SWS in which all the *n* MEs are identical and have two states with performance rates 0 and *g*, respectively is *k*-out-of-*r*-from-n system where $W = (r - k + 1)g$.

The introduction of the SWS model is motivated by the following examples.

1.1. Service system

Consider a conveyor-type service system that can process *r* incoming tasks simultaneously according to first-in-first-out rule and share a common limited resource. Each incoming task can have different states and the amount of the resource needed to process the task is different for each state of each task. The total resource needed to process *r* consecutive tasks should not exceed the available amount of

* Tel.: +972-4-864-9766; fax: +972-4-864-9780.
E-mail address: levitin@iec.co.il (G. Levitin).

0951-8320/02/$ - see front matter
PII: S0951-8320(02)00014-5

Nomenclature

n	number of MEs in SWS
r	number of consecutive MEs in SWS sliding window
W	minimal allowable cumulative performance of group of r consecutive MEs
F	SWS failure probability
R	SWS reliability
H_j	number of different states of ME j
g_{jh}	performance rate of ME j in state h
p_{jh}	probability of state h of ME j
Q_i	probability of state i of group of r consecutive MEs
$\mathbf{G}_i$	vector representing performance rates of group of r consecutive MEs in state i
$u_j(z)$	u-function representing PRD of jth ME
$U_m(z)$	vector-u-function representing PRD of mth group of r consecutive MEs
$\sigma(\mathbf{G})$	operator producing 1 if sum of vector $\mathbf{G}$ elements is less than W and 0 otherwise
Ω, Ψ	composition operators over u-functions
ϑ	operator over vector $\mathbf{G}$

Abbreviations

SWS	linear multi-state sliding window system
ME	multi-state element
PRD	performance rate distribution
UMGF, u-function	universal moment generating function

the resource. If there is no available resource to process r tasks simultaneously, the system fails.

1.2. Manufacturing

Consider a heating system that should provide certain temperature along a line with moving parts (Fig. 1). The temperature at each point of the line is determined by a cumulative effect of r closest heaters. Each heater consists of several electrical heating elements. The heating effect of each heater depends on the availability of its heating elements and therefore can vary discretely (if the heaters are different, the number of different levels of heat radiation and the intensity of the radiation at each level are specific to each heater). In order to provide the temperature, which is not less than some specified value at each point of the line; any r adjacent heaters should be in states where the sum of their radiation intensity is greater than an allowed minimum W.

It can be easily seen that the order of tasks arrival to the service system (first example) or allocation of heaters along a line (second example) can strongly affect the entire system reliability. Having a set of MEs, one can achieve considerable system reliability improvement by choosing proper elements ordering. In Ref. [7] Papastavridis and Sfakianakis first considered the optimal element allocation problem for consecutive k-out-of-r-from-n:F systems in which different elements can have different reliability. In this paper, the optimal element allocation problem is considered for the more general SWS model.

Section 2 of the paper presents the formal model description and the formulation of the optimization problem. Section 3 describes the technique used for evaluating the reliability of SWS with a given ME allocation. In Section 4, optimization technique is described. Illustrative examples are presented in Section 5.

2. Problem formulation

2.1. Assumptions

1. All n MEs of SWS are mutually independent.
2. Each ME j can be in one of H_j different states. Each state $h \in \{1, 2, \dots, H_j\}$ of ME j is characterized by its probability p_{jh} and performance rate g_{jh}. $\sum_{h=1}^{H_j} p_{jh} = 1$.
3. Each one of n MEs can be allocated at any one of n linearly ordered positions. Each position must contain one ME.
4. The SWS fails if the sum of performance rates of MEs located at any r consecutive positions is less than W.

In order to represent ME allocation in the SWS one can use an allocation function (vector) $\mathbf{C} = \{c(1), \dots, c(n)\}$ in which $c(j)$ is equal to number of ME allocated at position j. One can see that the total number of different allocation solutions (number of different vectors $\mathbf{C}$) is equal to $n!$ (the number of possible permutations in a string of n different numbers). For the set of MEs with a given

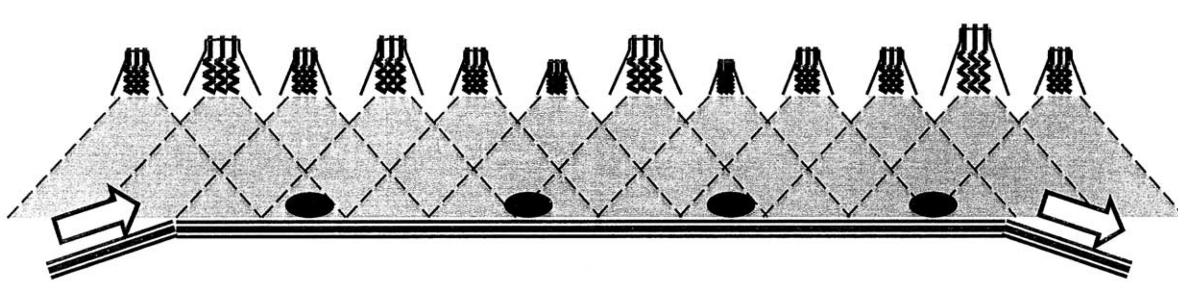

Fig. 1. Example of SWS with $r = 3$.