

# Optimal allocation of multi-state retransmitters in acyclic transmission networks

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## Abstract

In this paper, an algorithm for optimal allocation of multi-state elements (MEs) in acyclic transmission networks (ATNs) is suggested. The ATNs consist of a number of positions (nodes) in which MEs capable of receiving and sending a signal are allocated. Each network has a root position where the signal source is located, a number of leaf positions that can only receive a signal, and a number of intermediate positions containing MEs capable of transmitting the received signal to some other nodes. Each ME that is located in a nonleaf node can have different states determined by a set of nodes receiving the signal directly from this ME. The probability of each state is assumed to be known for each ME. The ATN reliability is defined as the probability that a signal from the root node is transmitted to each leaf node.

The optimal distribution of MEs with different characteristics among ATN positions provides the greatest possible ATN reliability. The suggested algorithm is based on using a universal generating function technique for network reliability evaluation. A genetic algorithm is used as the optimization tool. Illustrative examples are presented. © 2002 Elsevier Science Ltd. All rights reserved.

*Keywords:* Acyclic transmission network; Multi-state; Optimal allocation; Universal generating function

## 1. Introduction

Acyclic transmission networks (ATN) consist of a certain number of positions (nodes) in which multi-state elements (MEs) capable of receiving and/or sending signal are allocated. Each network has a root node where the signal source is located, a number of leaf nodes that can only receive a signal and a number of intermediate (neither root nor leaf) nodes containing MEs capable of transmitting the received signal to some other nodes. The signal transmission is possible only along links between the nodes. The networks are arranged in such a way that no signal leaving a node can return to this node through any sequence of nodes (no cycles exist).

Each ME located in nonleaf node can have different states determined by a set of nodes receiving the signal directly from this ME. The event that a ME is in a specific state is a random event. The probability of this event is assumed to be known for each ME and for every its possible state. All the MEs in the network are assumed to be statistically independent.

The whole network is in working condition if a signal from the root node is transmitted to each leaf node. Otherwise, the network fails. (Note that it is not always necessary for a signal to reach all the network nodes in order to provide its propagation to the leaf ones.)

An example of the ATN is a set of radio relay stations with a transmitter allocated at root node and receivers allocated at leaf nodes. Each station has retransmitters generating signals that can reach a set of next stations. Note that the composition of this set for each station depends on power and availability of retransmitter amplifiers as well as on signal propagation conditions.

The acyclic transmission network is a generalization of the tree-structured multi-state systems investigated by Malinowski and Preuss [1] and multi-state linear consecutively connected networks introduced by Hwang and Yao [2] and studied by Kossow and Preuss [3] and Zuo and Liang [4]. An algorithm for ATN reliability evaluation was suggested by Levitin in Ref. [5].

The problem of optimal ME allocation was first formulated by Malinowski and Preuss in Ref. [6] for linear consecutively connected system. In this problem, MEs with different characteristics should be allocated in system nodes in such a way that maximizes the system reliability. A multi-start local search algorithm was suggested for solving this problem.

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Abbreviations: ATN, acyclic transmission network; ME, multi-state element; UGF, universal generating function; GA, genetic algorithm.

### Nomenclature

$R$	ATN reliability
$N$	total number of nodes in ATN
$M$	number of leaf nodes in ATN
$D$	number of MEs to be allocated at ATN
$c_i$	$i$ th node of ATN
$\Delta$	set of MEs
$\Delta_i$	set of MEs allocated at $c_i$
$\lambda_{ik}$	set of nodes receiving a signal from ME located at $c_i$ when it is in state $k$
$K_i$	number of different states of individual ME located at node $c_i$
$\tilde{K}_i$	number of different states of group of MEs located at node $c_i$
$\hat{K}_i$	number of different states of group of MEs located at nodes $c_1, \dots, c_i$
$p_{i\lambda_{ik}}^d$	probability that a signal from $d$ th ME located at $c_i$ reaches set of nodes $\lambda_{ik}$
$\mathbf{P}^d$	state probability distribution matrix for ME $d$
$\mathbf{V}_i$	random binary vector representing set of ATN nodes receiving a signal directly from single ME located at node $c_i$
$\tilde{\mathbf{V}}_i$	random binary vector representing set of ATN nodes receiving a signal directly from group of MEs located at node $c_i$
$\hat{\mathbf{V}}_i$	random binary vector representing set of ATN nodes receiving a signal from $c_1$ through all the MEs located at $c_1, \dots, c_i$
$\mathbf{V}_{ik}$	value of $\mathbf{V}_i$ at state $k$ (vector representing the set $\lambda_{ik}$ )
$\tilde{\mathbf{V}}_{ik}$	value of $\tilde{\mathbf{V}}_i$ at state $k$
$\hat{\mathbf{V}}_{ik}$	value of $\hat{\mathbf{V}}_i$ at state $k$
$\tilde{q}_{ik}$	probability that $\tilde{\mathbf{V}}_i$ is equal to $\tilde{\mathbf{V}}_{ik}$
$\hat{q}_{ik}$	probability that $\hat{\mathbf{V}}_i$ is equal to $\hat{\mathbf{V}}_{ik}$
$u_{id}(z)$	$u$ -function corresponding to ME $d$ located at node $c_i$ (represents probabilistic distribution of $\mathbf{V}_i$ )
$\tilde{U}_i(z)$	$u$ -function corresponding to group of MEs located at node $c_i$ (represents probabilistic distribution of $\tilde{\mathbf{V}}_i$ )
$\hat{U}_i(z)$	$u$ -function corresponding to group of MEs located at nodes $c_1, \dots, c_i$ (represents probabilistic distribution of $\hat{\mathbf{V}}_i$ )
$\varphi$	$u$ -function simplification operator
$\Omega, \Psi$	composition operators over $u$ -functions
$\omega$	function for vector composition
$h(d)$	number of node in which ME $d$ is allocated (allocation function)

This paper presents an algorithm for optimal allocation of MEs in ATN. Simple extension of problem formulation [6] to ATN gives the following formulation.

Given ATN with  $N - M$  nonleaf nodes. Allocate  $D = N - M$  MEs in the nodes of the ATN (allowing only one

ME to be located in each node) in a way providing the maximal system reliability.

In many cases, even for  $D = N - M$ , greater reliability can be achieved if some of MEs are gathered in the same position providing redundancy (in hot standby mode) and some positions remain empty, than if all the MEs are evenly distributed between all the nonleaf nodes.

Consider, for example, the simplest case in which two identical MEs should be allocated within ATN with  $N = 3, M = 1$ . When allocated at node  $c_1$ , the MEs can have four states:

- total failure: ME does not connect node  $c_1$  with any other node (probability of this state is  $p_{1\emptyset}^1 = p_{1\emptyset}^2 = p_{1\emptyset}$ );
- ME connects  $c_1$  with  $c_2$  (probability of this state is  $p_{1\{2\}}^1 = p_{1\{2\}}^2 = p_{1\{2\}}$ );
- ME connects  $c_1$  with  $c_3$  (probability of this state is  $p_{1\{3\}}^1 = p_{1\{3\}}^2 = p_{1\{3\}}$ );
- ME connects  $c_1$  with both  $c_2$  and  $c_3$  (probability of this state is  $p_{1\{2,3\}}^1 = p_{1\{2,3\}}^2 = p_{1\{2,3\}}$ ).

When allocated at node  $c_2$ , the MEs can have two states:

- total failure: ME does not connect node  $c_2$  with any other node (probability of this state is  $p_{2\emptyset}^1 = p_{2\emptyset}^2 = p_{2\emptyset}$ );
- ME connects  $c_2$  with  $c_3$  (probability of this state is  $p_{2\{3\}}^1 = p_{2\{3\}}^2 = p_{2\{3\}}$ ).

Let us suppose that  $p_{1\emptyset} = p_{2\emptyset}$ . There are two possible allocations of the MEs within the ATN (Fig. 1):

- (A) both MEs are located in the first position;
- (B) the MEs are located in the first and second positions.

In case A, the ATN succeeds if at least one of the MEs is in state  $\{3\}$  or  $\{2, 3\}$  and the system reliability is

$$R_A = 2(p_{1\{3\}} + p_{1\{2,3\}}) - (p_{1\{3\}} + p_{1\{2,3\}})^2. \quad (1)$$

In case B, the ATN succeeds either when the ME located in the first position is in state  $\{3\}$  or  $\{2, 3\}$ , or if it is in state  $\{2\}$  and the second element is in state  $\{3\}$ . The system reliability in this case is

$$\begin{aligned} R_B &= p_{1\{3\}} + p_{1\{2,3\}} + p_{1\{2\}}p_{2\{3\}} \\ &= p_{1\{3\}} + p_{1\{2,3\}} + p_{1\{2\}}(1 - p_{2\emptyset}). \end{aligned} \quad (2)$$

Since  $p_{1\emptyset} = p_{2\emptyset}$ , one can rewrite expression (2) as

$$\begin{aligned} R_B &= p_{1\{3\}} + p_{1\{2,3\}} + (1 - p_{1\{3\}} - p_{1\{2,3\}} - p_{1\emptyset})(1 - p_{1\emptyset}) \\ &= 1 - 2p_{1\emptyset} + p_{1\emptyset}^2 + p_{1\emptyset}(p_{1\{3\}} + p_{1\{2,3\}}). \end{aligned} \quad (3)$$

By comparing Eqs. (1) and (3), one can decide which allocation of the elements is preferable for any given  $p_{1\emptyset}$  and  $p_{1\{3\}} + p_{1\{2,3\}}$ . Fig. 2 presents the decision curve  $R_A = R_B$  on the plane  $(p_{1\emptyset}, p_{1\{3\}} + p_{1\{2,3\}})$ . Observe that for combinations of  $p_{1\emptyset}$  and  $p_{1\{3\}} + p_{1\{2,3\}}$  located below the curve, the solution B is preferable while for combinations of

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