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Portfolio credit-risk optimization

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A B S T R A C T

This paper evaluates several alternative formulations for minimizing the credit risk of a portfolio of financial contracts with different counterparties. Credit risk optimization is challenging because the portfolio loss distribution is typically unavailable in closed form. This makes it difficult to accurately compute Value-at-Risk (VaR) and expected shortfall (ES) at the extreme quantiles that are of practical interest to financial institutions. Our formulations all exploit the conditional independence of counterparties under a structural credit risk model. We consider various approximations to the conditional portfolio loss distribution and formulate VaR and ES minimization problems for each case. We use two realistic credit portfolios to assess the in- and out-of-sample performance for the resulting VaR- and ES-optimized portfolios, as well as for those which we obtain by minimizing the variance or the second moment of the portfolio losses. We find that a Normal approximation to the conditional loss distribution performs best from a practical standpoint.

1. Introduction

For financial institutions, the benefits of managing (portfolio) credit risk include not only reduced monetary losses due to defaulted or downgraded obligations but also lower capital charges. While individual credit-risky positions can be hedged with credit derivatives such as credit default swaps, imperfectly correlated credit movements among counterparties also provide opportunities for mitigating credit risk at the portfolio level through diversification. In particular, the use of optimization techniques to restructure portfolios of credit-risky positions, is an attractive possibility. However, such procedures face numerous challenges, foremost being the difficulty of representing the portfolio credit loss distribution with sufficient accuracy. This paper formulates several alternative optimization problems that are derived from a structural (Merton) model of portfolio credit risk, and evaluates their effectiveness from the perspectives of risk mitigation and computational practicality.

Credit risk refers to the potential monetary loss arising from the default, or a change in the perceived likelihood of default, of a counterparty to a financial contract. Note that a reduction in the default probability, i.e., a transition to a more favorable credit state, results in a monetary gain. However, such gains are generally small relative to the losses that occur due to severe credit downgrades or default. Thus, the credit loss distribution (F) for a typical investment-grade portfolio is positively skewed, the long right tail being consistent with a small likelihood of substantial losses.

The complex relationships among asset prices, exposures and credit transitions preclude obtaining a closed-form representation of the actual credit loss distribution. Thus, for risk management purposes, it is necessary to replace F by some approximating distribution $\widehat{F}$. The form of $\widehat{F}$ varies depending on the underlying credit loss model. For example, reduced-form models (e.g., CreditRisk+ (Credit Suisse Financial Products, 1997)) provide $\widehat{F}$ in closed form. However, their underlying assumptions may be viewed as overly simplistic in that they fail to capture the effects of credit-state migrations and correlated movements of risk factors (de Servigny and Renault, 2004). In contrast, structural models (Gupton et al.,

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1997; Iscoe et al., 1999) can provide a more realistic representation but typically require $\widehat{F}$ to be an empirical distribution derived from Monte Carlo (MC) simulation.

Computing $\widehat{F}$ from Monte Carlo simulation presents challenges for assessing credit risk because common risk measures, such as Value-at-Risk (VaR)[1] and expected shortfall (ES), involve extreme quantiles in the right tail. Thus, obtaining accurate risk estimates requires a huge number of samples, or scenarios. Initial attempts at minimizing credit risk relied exclusively on MC simulation and included the full set of loss scenarios in the formulation (Mausser and Rosen, 2000, 2001; Andersson et al., 2001; Zenios and Jobst, 2001; Zagst et al., 2003). Clearly, a limitation of this approach is that the large size of the resulting optimization problem adversely affects computational performance. Subsequently, in Saunders et al. (2007) a large-portfolio approximation is used to obtain a more compact formulation.

Recently, variance-reduction techniques such as importance sampling (Tilke, 2006) have been proposed as a way to obtain stable optimal solutions, with a relatively small number of scenarios. However, a potential problem with importance sampling, is that the required shift in distribution depends on the portfolio's risk, which of course changes with the portfolio's composition during the course of the optimization. Thus, it is not clear that the shift induced by the initial portfolio is also effective for the optimal portfolio.

Structural models infer a counterparty's credit state from its associated creditworthiness index, which depends on systematic risk factors in the form of credit drivers as well as an idiosyncratic risk factor unique to each counterparty (Iscoe et al., 1999; Iscoe and Kreinin, 2000). Conditional on a set of values for the credit drivers, credit transitions for all counterparties are independent. This conditional independence property can be exploited to obtain $\widehat{F}$ in semi-analytical form, specifically, as an average of closed-form conditional loss distributions. Such representations are far more data-efficient than pure Monte Carlo sampling and the associated optimization problems are smaller as a result. In this paper, we evaluate the practicality of optimizing credit risk for three different representations of $\widehat{F}$:

- Monte Carlo sampling.
- A mixture of Normal (Gaussian) conditional loss distributions.
- A mixture of conditional mean (expected) losses.

For comparative purposes, we also consider the performance of variance-based formulations as a way of reducing a portfolio's VaR and ES. Variance minimization, which dates back to the seminal work of Markowitz (1952), remains in widespread use for risk management purposes. It is well known that minimizing variance has the effect of also minimizing VaR and ES only for elliptical (including Normal) distributions when the distribution's mean is constrained (McNeil et al., 2005). Thus, in our context, minimizing variance effectively assumes that $\widehat{F}$ is elliptical. This is likely to be a poor approximation to the actual portfolio credit loss distribution. Nevertheless, its popularity makes variance minimization a useful benchmark when evaluating the performance of the structurally based formulations. Since variance only measures dispersion around the mean, as a second, related benchmark we also minimize the second moment of the credit loss distribution (which takes the mean into account).

Our formulations and computational experiments are consistent with managing the risk of a banking book. Since a typical banking book may contain thousands of counterparties, we allow for optimizing over groups of counterparties. Thus, a portfolio manager might elect to assign all counterparties from a given industry to the same group, for example, and then use the results of the optimization to restructure the portfolio at the industry level. Such an approach is much more practical than enacting changes to a large number of individual contracts, as might be suggested by optimizing at the counterparty level. We also limit the amount of trading to what can be implemented reasonably when rebalancing the banking book; namely short positions are not permitted and new groups may not be added to the existing portfolio. These limitations are enforced only to provide a realistic assessment of the optimization results; nothing precludes relaxing or eliminating such restrictions from a formulational standpoint. Finally, although we account for credit migration, we assume that exposures are deterministic, i.e., positions are not marked to market.

The rest of this paper is organized as follows. In Section 2 we introduce some notation and basic concepts, and describe relevant input data for modeling credit-risky instruments. Section 3 introduces the structural model for portfolio credit risk, for which future credit events may be simulated. In Section 4, we describe several approximations for the loss distribution, F. Section 5 reviews the risk measures that we will optimize. The formulations of our credit-risk optimization problems follow in Section 6. We evaluate and analyze our computational results in Section 7. Finally, we present our conclusions, extensions and practical recommendations.

2. Portfolio credit losses

We are concerned with credit-risk modeling and optimization at the portfolio level only. Counterparty-level data is used as input to the portfolio-level models. Such data may be estimated from an internal model or provided by an external agency. We start by analyzing the input data for our portfolio-level credit-risk optimization problems.

In this paper, we only consider a single time period. At the end of the time period, each counterparty can migrate to a different credit state, resulting in our losses ℓ_c^j, where $j = 1, \ldots, N_{\mathrm{CP}}$ indexes counterparties and c indexes credit states. There are C credit states available for counterparties, enumerated from $c = 0$ (default) through increasing credit ratings, to the highest credit rating $c = C - 1$. Note that negative losses (gains) are incurred if a counterparty migrates to a more favorable state. The probability of being in the state c at the end of the time period is p_c^j, with p_0^j being the probability of default. Unadjusted exposures of counterparties at default correspond to their values ν^j ($\nu^j > 0$). In general, counterparty exposure is the economic loss that will be incurred on all outstanding transactions if a counterparty defaults, unadjusted by possible future recovery. Exposures are computed subject to netting, mitigation and collateral. The recovery at default is assumed to be deterministic and recovery-adjusted exposures are equal to $\nu^j(1 - \gamma^j)$, where γ^j is the recovery rate.

A portfolio consists of a number, N_{CP}, of counterparties grouped into N_{G} groups. For instance, counterparties from the same country, the same industry and having the same credit rating can be grouped together. Changing positions in groups is more practical as it allows altering just the between-groups positions, leaving the problem of within-group rebalancing as a lower level problem. From the point of view of modeling and optimization, grouping decreases the number of decision variables. Our modeling assumptions do not place any restrictions on the number of groups, group sizes or group compositions.

The value of the ith group, G_i, is $\nu_i = \sum_{j \in G_i} \nu^j$, and its loss is

[1] VaR is incorporated in the Basel II Capital Accord, which makes it an integral part of the regulatory requirements.