The Quarterly Review of Economics and Finance 54 (2014) 393–404

Contents lists available at ScienceDirect

The Quarterly Review of Economics and Finance

journal homepage: www.elsevier.com/locate/qref

# Multifactor risk loadings and abnormal returns under uncertainty and learning

Simone Salotti [a], Carmine Trecroci [b,*]

[a] Department of Accounting, Finance and Economics, Oxford Brookes University, Oxford, OX33 1HX, UK
[b] Department of Economics and Management, University of Brescia, Via San Faustino 74/b, 25122 Brescia, Italy

ARTICLE INFO

*Article history:*
Received 2 July 2012
Received in revised form 14 January 2014
Accepted 11 April 2014
Available online 9 May 2014

*JEL classification:*
G12
G31
C51

*Keywords:*
Multifactor models
Time-varying alphas
Time-varying betas

ABSTRACT

We explore the time variation of factor loadings and abnormal returns in the context of a four-factor model. Our methodology, based on an application of the Kalman filter and on endogenous uncertainty, overcomes several limitations of competing approaches used in the literature. Besides taking learning into account, it does not rely on any conditioning information, and it only imposes minimal assumptions on the time variation of the parameters. Our estimates capture both short- and long-term fluctuations of risk loadings and abnormal returns, also showing marked variation across US industry portfolios. The results from mean-variance spanning tests indicate that our baseline model yields accurate predictions and can therefore improve pricing and performance measurement.



## 1. Introduction

Do systematic risks vary over time and across industries? Multifactor asset pricing models posit a linear relationship between asset returns and various risk factors that reflect the impact of market conditions on beliefs and/or preferences, and hence on risk premia. Motivated by empirical evidence on market anomalies, the model by Fama and French (1992) (see also Carhart, 1997) added two factors to the conventional form of the CAPM: market capitalization (size) and book-to-market (value/growth). More recent literature shows that expected returns and their key drivers, i.e., risk premia and risk loadings (betas), are likely to experience some variation over time (Ang & Chen, 2007; Cooper & Priestley, 2009; Fama & French, 1997, 2006; Guidolin & Timmermann, 2008a; Lewellen & Nagel, 2006). Most existing contributions account for that by allowing betas to change, while constant abnormal returns (alphas) are extracted via numerical optimization. Three main competing approaches have emerged to model such dynamics. Lewellen and Nagel (2006) and Fama and French (2006) employ rolling window OLS regressions. Alternatively, Ferson and Harvey (1999) and Avramov and Chordia (2006) impose parametric relationships between risk loadings and a set of (macroeconomic and financial) variables proxying for the state of the economy, i.e., they use an instrumental variables approach. Ghysels and Jacquier (2006), Ouysse and Kohn (2010) and Guidolin, Ravazzolo, and Tortora (2013) provide compromises between these two routes. Guidolin and Timmermann (2008a) and Abdymomunov and Morley (2011) explore further methods based on regime-switching processes.[1]

In all cases, results are far from conclusive. First, the rolling-window approach imposes an exogenous timing to changes in the market risk premium, with results depending crucially on the choice of the window length. It also tends to understate the variance of the true betas (Ang & Chen, 2007). Second, in most approaches and particularly with conditioning information/instrumental variables, models tend to be richly parameterized and often rely on strict priors about time variation in the mean and volatility of the conditional risk premia. In addition, idiosyncratic risk is rarely allowed to vary over time while

[1] The latter contribution, however, posits strong assumptions on the volatility regimes. Guidolin and Timmermann (2008b) find evidence of two regimes in the distribution of international stock returns. Their model allows investor preferences to depend also on skewness and kurtosis, while the exposures to risk factors as well as the price of covariance, co-skewness, and co-kurtosis risk vary across regimes.

* Corresponding author. Tel.: +39 030 2988812.
*E-mail addresses:* ssalotti@brookes.ac.uk (S. Salotti), trecroci@eco.unibs.it (C. Trecroci).

http://dx.doi.org/10.1016/j.qref.2014.04.003
1062-9769/

results from these models have been proven to be highly sensitive to the choice of the instrumental variables (Harvey, 2001). Annaert and van Campenhout (2007) and Trecroci (2013) test for structural breaks, finding strong evidence against the hypothesis of constant exposures. Lastly, Ghysels (1998) shows that simple constant betas outperform several parametric beta models, while Jostova and Philipov (2005) and Ang and Chen (2007) find substantial evidence of inconsistence in CAPM constant-coefficients and rolling regressions. This might be due at least partly to the fact that tests based on parametric approaches are strictly valid only if the econometrician knows the full set of state variables available to investors. That is a fairly strong assumption, but even taking it as valid, the complexity of the structural relationships would make their direct estimation unfeasible.

In practice, several real-world factors beyond the appreciation of the econometrician are likely to play a significant role in the determination and evolution of risk loadings. One of them is investor uncertainty. Presumably, investors forecast risk loadings and risk premia through some complex learning process that reflects uncertainty about their distributional properties. Changes in the structure of the economy and in financial markets make it reasonable to think about, and therefore model, risk sensitivities as time-varying quantities, particularly over long samples and at business-cycle frequencies. In this paper, we provide time-varying estimates of alphas and betas that are derived under very general assumptions about the investors' information set.

We estimate a time-series specification of a four-factor model, based on time-varying alphas, risk loadings and idiosyncratic risk, for ten US industry-specific portfolios. The key contribution of this paper lies in the fact that we obtain estimates of risk components that are endogenous with respect to uncertainty. Operationally, uncertainty is defined as the conditional error variance of the optimal forecast of alphas and betas. This setting replicates the learning activity of rational investors, who must infer the risk loadings from available information and optimally update them as new information becomes available. Cognitive limitations and/or shortage of degrees of freedom are likely to force investors to under-parameterize their forecasting models. Accordingly, we posit that changes in monthly factor returns fully reflect the arrival of relevant information. This parsimonious model allows for changes in perceived risks due to factors unobserved by the econometrician, such as shifts in the quantity of undiversifiable risk, which might be learning-induced. The time variation in the parameters is captured through an application of the Kalman filter (KF, henceforth) that yields monthly alpha and beta time series.

The parsimonious methodology in this paper offers several benefits over the existing approaches. First, it does not rely on any instrumental variables/conditioning information to identify the source of time variation of the estimated parameters. Second, it yields portfolio sensitivities to risk factors that change by mimicking the agents' learning process and taking uncertainty into account. Third, it estimates jointly each period's conditional alphas and betas by only imposing minimal assumptions about their period-to-period variation. Fourth, our method is based on time series regressions, which overcome the limitations of a full cross-sectional approach (Lettau & Ludvigson, 2001). Fifth, it requires narrow parameterization compared to alternative approaches such as multi-equation settings, or other state-space models with regime switching. While the latter models (for instance, see Guidolin & Timmermann, 2008a, 2008b) allow for a more intuitive treatment of abrupt shifts in volatility, they impose assumptions on the dynamics of alphas and betas that are more restrictive than those of our structural time series approach. Finally, we also improve on more complex Bayesian methodologies, like Jostova and Philipov (2005); the latter is the only previous attempt to allow for the joint time variation of the alphas and betas. However, these authors focus on a one-factor model.

The main contributions of our paper are the following. Our estimates of the four-factor model reveal that abnormal returns and risk loadings experience considerable fluctuations over time, along patterns that are clearly different across diversified industry portfolios.[2] This confirms that investors update their forecasts on a more frequent and systematic basis than existing analyses entertain. In addition, we find evidence of persistent non-zero excess returns for portfolios with zero exposures to the risk factors. Although business conditions might be viewed as the key drivers of changes in asset risk, our results suggest that using them to model the risk factors (as it is done in the instrumental variables approach) may impose too strong restrictions/assumptions than allowed by the data. We also find that stationary betas imply larger pricing errors than nonstationary ones. This evidence underlines the nature of betas as functions of macroeconomic shocks that can have permanent effects. For instance, over the past two decades stocks have experienced a widespread increase in the variability of their exposure to fundamental risks. Finally, a novel extension of mean-variance spanning tests in our time-varying context yields evidence that our portfolios improve on the investment opportunities represented by the conventional four-factor model. In addition, the test results point to a substantial increase of uncertainty and systematic risks in the run-up to the recent financial crisis.

The rest of the paper is structured as follows. Section 2 explains our methodology and how it accounts for the learning problem of investors under uncertainty. Section 3 presents estimates of time-varying alphas, betas and pricing uncertainty and performs a test of their predictive ability, whereas Section 4 subjects them to tests of mean-variance spanning. Section 5 concludes.

## 2. A parsimonious representation with uncertainty, learning and time variation

The time-varying-parameters, Kalman-filter-based method (TVK, henceforth) we employ in this paper accounts for two sources of uncertainty: one associated with future idiosyncratic risk, and one arising because of the evolution of risk loadings. Conditional uncertainty is therefore directly tied to observed returns, which contain the relevant information for investment choices. This framework proxies for a more complex environment, in which investors face uncertainty about their model specification and choose parsimonious trading strategies. Consequently, the model allows for time variation in both the mean and the homoskedastic stochastic components of the alpha and beta processes. The resulting TVK estimates depend only on portfolio and market returns.

Our empirical analysis is based on the following four-factor model:

$$R_t^{ei} = \alpha^i + \beta^i R_t^{eM} + s^i R_t^{SMB} + h^i R_t^{HML} + m_t^i R_t^{UMD} + \varepsilon_t^i, \quad (1)$$

where $R_t^{ei}$ is the return on test asset $i$ in excess of the one-month Treasury bill rate; $R_t^{eM}$ is the excess return on the market; $R_t^{SMB}$, $R_t^{HML}$ and $R_t^{UMD}$ are the returns on the SMB, HML and momentum factor portfolios, respectively; $\beta^i$, $s^i$, $h^i$ and $m^i$ are the asset's factor loadings. It is widely believed that the dynamics of fundamental risk factors, such as the market, growth opportunities, financial distress, as well as the firm's size, drive the cross-section of risks and returns (see, e.g., Cochrane, 2005). $R_t^{SMB}$ is included in the model

[2] Here we do not deal with the issue of alphas and betas predictability. Trecroci (2013) studies the correlation of one-factor TVK alphas and betas with various indicators of the business cycle and market conditions.