Applied Soft Computing 38 (2016) 890–906

Contents lists available at ScienceDirect

Applied Soft Computing

journal homepage: www.elsevier.com/locate/asoc

Credibilistic multi-period portfolio optimization model with bankruptcy control and affine recourse

Yong-Jun Liu, Wei-Guo Zhang*, Qun Zhang

School of Business Administration, South China University of Technology, Guangzhou 510641, PR China

ARTICLE INFO

Article history:
Received 26 March 2013
Received in revised form 10 June 2015
Accepted 15 September 2015
Available online 28 October 2015

Keywords:
Multi-period portfolio optimization
Credibility measure
Bankruptcy control
Hybrid PSO algorithm

ABSTRACT

Avoiding the possibility of bankruptcy during the investment horizon is very important to multi-period portfolio management. This paper considers a multi-period fuzzy portfolio selection problem with bankruptcy control. A multi-period portfolio optimization model imposed by a bankruptcy control constraint in fuzzy environment is proposed on the basis of credibility theory. In the proposed model, a linearly recourse policy is used to reflect the influence of historical predication basis on current portfolio decision. Three optimization objectives, viz., maximizing the terminal wealth and minimizing the cumulative risk and the cumulative uncertainty of the returns of portfolios over the whole investment horizon, are taken into consideration. For solving the proposed model, a fuzzy programming approach is applied to transform it into a single objective programming model. Then, a hybrid particle swarm optimization algorithm is designed for solution. Finally, an empirical example is presented to illustrate the application of the proposed model and solution comparisons are also given to demonstrate the effectiveness of the designed algorithm.

1. Introduction

One of the basic principles of financial investment is diversification where investors should allocate their capital among different assets according to their own preferences. Markowitz [37] originally proposed the well-known mean-variance (MV) model, which laid a fundamental basis for single-period portfolio selection. However, in real life, investors tend to construct multi-period portfolios and they usually need to rebalance their positions from time to time. After Markowitz's pioneering work, numerous researchers extended his work to multi-period case, see for instance in Elton and Gruber [14], Hakansson [20] and Mossin [39].

Nowadays, there has been a continuing effort in extending portfolio selection from single-period case to multi-period case by using different approaches. Li and Ng [31] used the same approach to handle the continuous-time setting portfolio selection problem by using the idea of embedding the problem in a tractable auxiliary problem. Then, they obtained breakthrough result, that is, the optimal mean-variance portfolio policy and the corresponding efficient frontier. Cui et al. [11] proposed a mean-variance model for multi-period portfolio selection problem with no shorting constraint. Li et al. [32] presented a dynamic mean-variance model by taking no-shorting constraints into consideration and obtained the efficient frontier and the efficient investment strategies for original mean-variance problem. Yan et al. [53] proposed a class of multi-period semi-variance model and designed a novel hybrid genetic algorithm (GA) with particle swarm optimization (PSO) for solving the proposed model. Calafiore [3] investigated multi-period sequential decision problems for financial asset allocation and proposed a multi-period portfolio selection model with the objective of minimizing a cumulative risk measure over the investment horizon, while satisfying portfolio diversity constraints at each period and achieving or exceeding a desired terminal expected wealth target. Costa and Araujo [10] considered a multi-period generalized mean-variance model with Markov switching in the key market parameters. Several multi-period portfolio optimization models in stochastic market where consists of a riskless asset and several risky assets were explored by Çelikyurt and Özekici [5]. In [22], a multi-period stochastic optimization model was formulated for dynamic asset allocation.

As we know, in multi-period investment setting, a bankruptcy may occur when the total wealth of an investor falls below a predetermined "disaster" level in any intermediate or the final time period. Thus, how to control the occurring of bankruptcy in multi-period portfolio selection has become a hot topic in financial risk management. Nowadays, some researchers have investigated the bankruptcy control problems for multi-period portfolio selection under the framework of probability theory, see for example Li and Li [29], Wei and Ye [52] and Zhu et al. [59]. Note that above-mentioned bankruptcy control models used variance as risk measure. However, due to make no distinction between gains and losses, variance risk measure has a distinguished drawback. Specially, when probability distributions on the rates of return on risky assets are asymmetric, variance becomes a deficient risk measure because it may have a potential danger to sacrifice too much expected return in eliminating both low and high return extremes. Yu et al. [55] substituted the classical variance with the absolute deviation as risk measure and proposed a dynamic portfolio optimization model with risk control. Though, it can simplify the computation by using absolute deviation as risk measure. It still cannot distinguish gain and loss. Besides, transaction costs is not incorporated into these bankruptcy control literatures mentioned above, which may lead to an inefficient portfolio. So it is necessary to present a more realistic risk control model for multi-period portfolio selection.

* Corresponding author. Tel.: +86 208711421; fax: +86 208711405.
E-mail addresses: yjlgx0202@126.com, liuyongjun0202@aliyun.com (Y.-J. Liu), wgzhang@scut.edu.cn, zhwg61@263.net (W.-G. Zhang).

http://dx.doi.org/10.1016/j.asoc.2015.09.023

Most of literatures were formulated on the basis of probability theory. Though probability theory is one of the main tools for handling the uncertainty in finance, it cannot describe uncertainty completely since there exist many non-probabilistic factors in financial markets such as economic, social, political and people's psychological factors, etc. Thus, the fuzzy uncertainty associated with financial markets cannot be neglected. With the widely use of fuzzy set theory in Zadeh [56], more and more researchers have realized that they could use fuzzy set theory to handle the fuzziness, vagueness or ambiguity in financial markets such as Alimi et al. [1], Ghaffari-Nasab et al [16] and Gharakhani and Sadjadi [17]. Actually, fuzzy portfolio selection problem was researched from 1990s. Based on possibility theory, numerous portfolio selection models had been proposed, e.g., Carlsson et al. [4], Deng and Li [12], Tanaka and Guo [47] and Zhang et al. [58]. Due to the self-dual of possibility measure, Liu and Liu [34] defined a self-dual measure (i.e., credibility measure) to quantify the chance of occurrence of fuzzy events. After that, numerous researchers proposed several portfolio selection models based on credibility measure such as, mean-variance model [23], mean-semivariance model [24], mean-variance adjusting model [57] and so on. Though, various portfolio selection models with fuzzy returns have been proposed. Most of them are single-period portfolio selection models. The research about multi-period portfolio selection in fuzzy environment is still in exploration phase. In 2011, Sadjadi et al. [42] investigated a fuzzy multi period portfolio selection problem with different borrowing and lending rates. Liu et al. [36] and Zhang et al. [63] discussed multi-period portfolio selection problems in fuzzy uncertain economic environment by using possibility theory. Zhang and Liu [62] studied multi-period portfolio selection problem with bankruptcy control in fuzzy environment based on credibility measure. To our knowledge, few researches have considered the influence of historical predication bias on current portfolio decision for multi-period fuzzy portfolio selection problem with bankruptcy control. The purpose of this paper is to discuss the bankruptcy control problem with feedback control policies for multi-period fuzzy portfolio selection. Different from Zhang and Liu [62], we use credibilistic lower absolute deviation as risk measure. Meanwhile, we consider the effect of the uncertainty associated with fuzzy returns on risky assets and use credibilistic entropy to measure them. In addition, we apply linear recourse policies to reflect the control action of historical predication bias on current decision. We propose a credibilistic multi-period portfolio optimization model with bankruptcy control and affine recourse.

The remainder of this paper is organized as follows. For the better understanding of the paper, we introduce some basic definitions about fuzzy variables in Section 2. In Section 3, we formulate a credibilistic multi-period portfolio optimization model with bankruptcy control and affine recourse. In Section 4, we first apply the fuzzy programming approach in Zimmermann [61] to transform the proposed model into a single-objective optimization problem. Then, we design a hybrid particle swarm optimization (PSO) to solve it. In Section 5, we give an empirical example to illustrate the application of our model. Meanwhile, solution algorithm comparisons are also given to demonstrate the effectiveness of the designed algorithm. Finally, we conclude the paper in Section 6.

2. Preliminaries

In this section, let us first briefly review some concepts of credibility theory, which we need in the following sections. Let ξ be a fuzzy variable with membership function $\mu(x)$ and u be a real number. Then, the credibility of $\{\xi \le u\}$ is defined as (see Liu and Liu [34])

$$\mathrm{Cr}\{\xi \le u\} = \frac{1}{2}\left(\sup_{x \le u}\mu(x) + 1 - \sup_{x > u}\mu(x)\right), \tag{1}$$

where credibility measure Cr is self-dual, i.e., $\mathrm{Cr}\{\xi \le u\} + \mathrm{Cr}\{\xi \ge u\} = 1$. When the credibility value of a fuzzy event achieves 1, it means that the fuzzy event will surely happen. Fuzzy events with credibility values will lead to different occurring chances. The fuzzy event with higher credibility value will have more chance to happen. In this paper, we use credibility measure to quantify the chance of a fuzzy event occurring.

For example, if $\xi = (a, \alpha, \beta)$ is a triangular fuzzy number, its membership function $\mu_\xi(x)$ can be expressed as follows (see Fig. 1(a))

$$\mu_\xi(x) = \begin{cases} \dfrac{x-(a-\alpha)}{\alpha}, & \text{if } \ a-\alpha \le x \le a, \\[2mm] \dfrac{a+\beta-x}{\beta}, & \text{if } \ a \le x \le a+\beta, \\[2mm] 0, & \text{otherwise.} \end{cases}$$

From Fig. 1, the following results can be obtained. If $\mu_\xi(x) = 1$, then x equals to ξ with degree of membership one. If $\mu_\xi(x) = 0$, then x belongs to ξ with degree of membership zero (i.e., $\xi \notin (a-\alpha, a+\beta)$, ξ is too far from a). If $\mu_\xi(x) \in (0, 1)$, then x belongs to ξ with an intermediate degree of membership (i.e., is close enough to a).

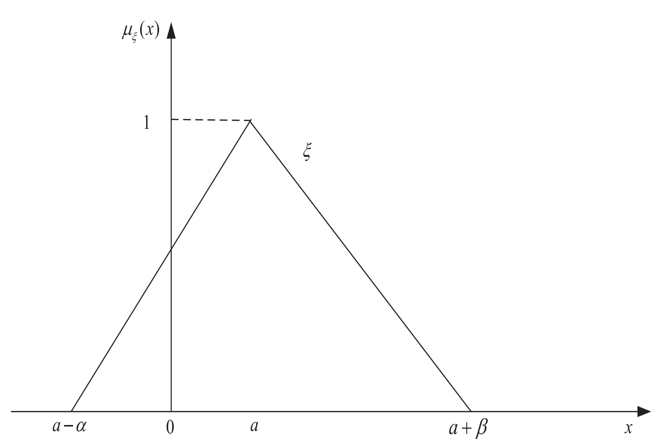

Fig. 1(a). Triangular membership function.