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Alternative bankruptcy prediction models using option-pricing theory

Andreas Charitou [a], Dionysia Dionysiou [b,*], Neophytos Lambertides [c], Lenos Trigeorgis [d]

[a] Department of Accounting and Finance, University of Cyprus, P.O. Box 20537, Nicosia CY 1678, Cyprus
[b] Division of Accounting and Finance, University of Stirling, Stirling FK9 4LA, UK
[c] Department of Commerce, Finance and Shipping, Cyprus University of Technology, 140 Ayiou Andreou Street, 3603 Lemesos, Cyprus
[d] Bank of Cyprus Chair Professor of Finance, Department of Public and Business Administration, University of Cyprus, P.O. Box 20537, Nicosia CY 1678, Cyprus

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ABSTRACT

We examine the empirical properties of the theoretical Black–Scholes–Merton (BSM) bankruptcy model. We evaluate the predictive ability of various existing modifications of the BSM model and extend prior studies by estimating volatility directly from market-observable returns on firm value. We show that parsimonious models using our direct market-observable volatility estimate perform better than alternative, more sophisticated, models. Our findings suggest the adoption of simpler modelling approaches relying on market data when implementing the BSM model.

1. Introduction

In recent years, especially during the ongoing economic crisis, increased attention has been paid to default models. The Black–Scholes–Merton (BSM) default prediction model has been one of the most influential and widely used models in corporate finance in the past four decades. It has been widely used to investigate, *inter alia*, default probabilities and recovery rates (e.g., Brunche and González-Aguado, 2010), default risk and returns (e.g., Chava and Purnanandam, 2010; Da and Gao, 2010; Li and Miu, 2009; Garlappi et al., 2008; Vassalou and Xing, 2004), default risk and executive compensation (Kadan and Swinkels, 2008) and default correlations and determinants (e.g., Lando and Nielsen, 2010; Campbell et al., 2008).

Various attempts have been made over the years to modify and extend the BSM model. Prominent among these are the KMV–Merton approach and the Bharath and Shumway (2008) (BhSh) model. The aim of this study is to evaluate the predictive performance of alternative specifications of the BSM default model and test our hypothesis (extending an insight recognized by BhSh) that it is not necessary to estimate firm value and its volatility by solving the simultaneous BSM equations (or other iterative procedures) to achieve accurate forecasts of default probabilities. We show that when volatility is estimated directly from observable firm value return data rather than from intermediate estimations of other observable variables, the BSM model performs better than previous alternative modifications.

BhSh have re-examined the specification and forecasting ability of the KMV–Merton approach. This latter specification recognizes that default may be triggered by the firm's inability to meet intermediate scheduled debt payments before maturity, a feature ignored by the original BSM model. Actually, KMV–Merton uses more complex iterative procedures than BSM to estimate the firm value and its volatility, as it recognizes that solving the BSM simultaneous equations may provide inaccurate estimations and biased probabilities of default when market leverage is volatile (Crosbie and Bohn, 2003). Moreover, the KMV–Merton approach does not rely on the cumulative normal distribution function to estimate the default probabilities. Instead, it relies on a large proprietary dataset and past experience to estimate the probability of intermediate default, with a downward adjustment of the default boundary trigger at debt maturity. For these reasons, exact replications of the KMV–Merton approach are *not* feasible. Various studies following KMV–Merton procedures sometimes have different assumptions and inputs. We

* Corresponding author. Tel.: +44 (0) 1786467292; fax: +44 (0) 1786467308.
E-mail addresses: charitou@ucy.ac.cy (A. Charitou), dionysia.dionysiou@stir.ac.uk (D. Dionysiou), n.lambertides@cut.ac.cy (N. Lambertides), lenos@ucy.ac.cy (L. Trigeorgis).

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refer to common versions of the KMV–Merton model as "feasible" KMV–Merton approach.[1]

BhSh suggest a specific KMV–Merton measure based on a "naive" approach, in which option variables are observable by the market: the firm value can be estimated directly as the sum of the market value of equity and the face value of debt. Volatility is then estimated based on a specific relation between the volatilities of debt and equity.[2] BhSh argue that their measure produces more accurate predictions than previous models and that it is not necessary to solve the two simultaneous equations for estimating firm value and its volatility for valid forecasting inferences.

We conjecture, though, that since the BhSh measure is derived from intermediate data (i.e., data extracted from iterative procedures without access to proprietary datasets), it might be less effective than a more parsimonious, market based volatility estimation (i.e., a volatility estimation that derives directly from historic firm value return data). Intermediate estimations may generally limit the model's predictive ability for two reasons: (i) numerical procedures typically used to solve the BSM simultaneous equations can be subject to convergence problems (Crosbie and Bohn, 2003), and (ii) when ad hoc procedures are used based on a specific sample (as in Bharath and Shumway, 2008), they may be subject to sample bias when applied by future users using different samples.

We test the predictive ability of the default measure based on our direct volatility estimation and compare it with that based on the BhSh volatility estimation as well as alternative model specifications, such as the "feasible" KMV–Merton approach and Charitou and Trigeorgis' (2004) intermediate default model. By applying Cox proportional hazard models to large datasets of bankrupt and healthy US firms over the period 1985–2009, we test whether alternative specifications of the option variable estimations used in the literature (e.g., different volatility estimations, alternative time-to-maturity proxies and different default trigger boundaries) lead to measures with improved ability to forecast default probabilities. To do this, we examine the forecasting ability of alternative model specifications that incorporate combinations of suggested modifications. We then compare the competing model specifications according to the area under the ROC curve (see *inter alia* Vassalou and Xing, 2004; Agarwal and Taffler, 2008) and according to their ability to predict bankruptcy based on the decile method (see Shumway, 2001; Bharath and Shumway, 2008).

The results verify the main insight noted by Bharath and Shumway (2008) and extended by us in directly estimating volatility from market firm data, confirming that it is not necessary to solve the BSM simultaneous equations to accurately predict default. Our findings further indicate that models with default measures estimated based on all market-available data generally have improved ability to forecast default. This result is robust to alternative specifications based on ROC curve comparisons and decile approach tests. Our direct market-based volatility measure results in a more accurate BSM specification. It is theoretically sound and avoids reliance on an ad hoc estimation that could be subject to sample properties of prior studies. As proprietary information and ad hoc relations are not necessary, our specification can more readily be applied in practice. Thus, it has fewer implementation issues compared to prior studies (see later footnote 7 for potential limitations). Finally, we suggest that more parsimonious specifications with direct and theoretically sound measures of time-to-maturity and default boundaries might be superior in forecasting default, especially when access to proprietary information is not available. We contribute to the extant literature by suggesting a parsimonious and more effective direct volatility estimation using market-available data.

The following section describes the theoretical framework based on Black–Scholes–Merton (1973, 1974, 1977), the "feasible" KMV–Merton approach and the methodology employed by Bharath and Shumway (2008). Section 3 describes our sample and methodology for comparing these models and incorporating our direct volatility estimate. Section 4 discusses the empirical results and robustness tests. Section 5 concludes.

2. Alternative option-pricing models of business default

2.1. The Black–Scholes–Merton (BSM) model

Under the BSM model, firm value V_t is assumed to follow standard Geometric Brownian motion, $dV_t/V_t = (\alpha - D)dt + \sigma_V dz$, where α represents the total expected rate of return on firm value, D is the total firm payout (as% of V), σ_V the standard deviation of firm value returns and dz an increment of a standard Wiener process. Following Merton (1974, 1977), any claim F whose value is contingent on a traded asset with value V, having a payout or yield D% and time-to-maturity T, must satisfy the fundamental partial differential equation (p.d.e.):[3]

$$\frac{1}{2}\sigma_v^2 V^2 \frac{\partial^2 F}{\partial V^2} + (r - D)V\frac{\partial F}{\partial V} + \frac{\partial F}{\partial t} - rF = 0 \quad (1)$$

where r is the risk-free interest rate. Solution to the above p.d.e. under appropriate boundary conditions is given by the Black–Scholes formula viewing the equity of the firm E as a European call option on a dividend-paying asset (firm asset value) V with exercise price the face value (principal) of the debt B due at debt maturity T:

$$E(V, T) = Ve^{-DT}N(d_1) - Be^{-rT}N(d_2) \quad (2)$$

$$\text{where} \quad d_1 = \frac{\ln(V/B) + (r - D + 0.5\sigma_V^2)T}{\sigma_V\sqrt{T}} \quad (3)$$

$$\text{and} \quad d_2 = d_1 - \sigma_V\sqrt{T} = \frac{\ln(V/B) + (r - D - 0.5\sigma_V^2)T}{\sigma_V\sqrt{T}} \quad (4)$$

Here E = European call option (firm equity), B = face value (principal) of the debt, V = value of the firm (assets), D = total payout yield (constant), σ_V = standard deviation of firm value changes (returns in V), T = time-to-debt maturity, r = risk-free interest rate, and $N(d)$ = cumulative standard normal distribution function (from $-\infty$ to d). The term $Ve^{-DT}N(d_1)$ represents the discounted expected value of the firm if it is solvent. Be^{-rT} is the present value of the principal debt amount B due at maturity T, and $N(d_2)$ is the (risk-

[1] Publications by authors affiliated with KMV (e.g., Crosbie and Bohn, 2003; Arora et al., 2005) suggest that KMV-Merton is not a single model, as KMV makes frequent modifications and adjustments to its procedures. Herein, the term KMV-Merton is used to denote a "feasible" KMV-Merton approach as commonly applied in prior studies i.e., time-to-default equals one, default boundary equals the current liabilities plus half of long-term liabilities, and firm value and its volatility derive following the BSM algorithms, involving solution of the two Merton simultaneous equations (e.g., see Bharath and Shumway, 2008). Other studies offer improvements to the BSM model and "feasible" KMV-Merton approach by accounting for intermediate default via a liquidity or cash coverage ratio (e.g., Charitou and Trigeorgis, 2004), by using accounting default models (e.g., Agarwal and Taffler, 2008; Hillegeist et al., 2004) or by estimating a distance-to-default measure based on a volatility-adjusted measure of leverage (e.g., Duffie et al., 2007).

[2] Volatility of debt is obtained from iterative procedures, being equal to 0.05 plus 25% of the volatility of equity. Firm asset value volatility is the weighted average volatility (based on the weights of debt and equity) of the volatilities of debt and equity.

[3] We adjust the formulas for dividend yield, D, although the original BSM model assumes $D = 0$.