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On the investment implications of bankruptcy laws ☆

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ABSTRACT

Axiomatic analysis of bankruptcy problems reveals three major principles: (i) *proportionality* (*PRO*), (ii) *equal awards* (*EA*), and (iii) *equal losses* (*EL*). However, most real life bankruptcy procedures implement only the proportionality principle. We construct a noncooperative investment game to explore whether the explanation lies in the alternative implications of these principles on investment behavior. Our results are as follows (i) *EL* always induces higher total investment than *PRO* which in turn induces higher total investment than *EA*; (ii) *PRO* always induces higher egalitarian social welfare than both *EA* and *EL* in interior equilibria; (iii) *PRO* induces higher utilitarian social welfare than *EL* in interior equilibria but its relation to *EA* depends on the parameter values (however, a numerical analysis shows that on a large part of the parameter space, *PRO* induces higher utilitarian social welfare than *EA*).

1. Introduction

Following the seminal work of O'Neill (1982), a vast literature focused on the axiomatic analysis of "bankruptcy problems". As the name suggests, a canonical example to this problem is the case of a bankrupt firm whose monetary worth is to be allocated among its creditors. Each creditor holds a *claim* on the firm and the firm's liquidation value is less than the total of the creditors' claims. The axiomatic literature provided a large variety of "bankruptcy rules" as solutions to this problem. The most central of these rules are all based on one (or more) of three central principles: (i) **proportionality**, (ii) **equal awards**, and (iii) **equal losses**.[1]

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[1] As their names suggest, these principles suggest that the agents' shares should be chosen, respectively, (i) proportional to their investments, (ii) so as to equate their awarded shares, (iii) so as to equate their losses from initial investment. There are bankruptcy rules purely based on one of these principles (such as the Proportional, Constrained Equal Awards, Constrained Egalitarian, Constrained Equal Losses rules) as well as rules that apply different principles on different types of problems (such as the Talmud rule which uses both equal awards and equal losses principles).

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Bankruptcy has also been a central topic in corporate finance where researchers analyze a large number of issues related to it (e.g. see Hotchkiss et al., 2008).[2] This literature shows that, in practice almost every country uses the following rule to allocate the liquidation value of a bankrupt firm.[3] First, creditors are sorted into different priority groups (such as secured creditors or unsecured creditors). These groups are served sequentially. That is, a creditor is not awarded a share until creditors in higher priority groups are fully reimbursed. Second, in each priority group, the shares of the creditors are determined in *proportion* to their claims.[4]

In this paper, we explore why in actual bankruptcy laws, *proportionality* has been preferred over the other two principles. Our starting observation is that alternative bankruptcy rules affect investment behavior in different ways. More formally, each rule induces a different noncooperative game among the investors. Comparing the equilibria of these games, in terms of total investment or social welfare, might provide us ways of comparing alternative bankruptcy rules and thus, the principles underlying them, in a way that is not previously considered in either the axiomatic literature or the corporate finance literature on bankruptcy, both discussed at the end of this section.

As a representation of the proportionality principle, we use the *Proportional rule* (hereafter, *PRO*), which assigns each investor a share proportional to his investment. We then look at a class of rules that mix the proportionality principle with equal awards (hereafter, $AP[\alpha]$). These rules pick an α-weighted average of the proportional allocation and the (pure) equal division. For $\alpha = 0$, the rule $AP[\alpha]$ coincides with an "unconstrained equal awards rule" (*EA*) which always chooses equal division. For $\alpha = 1$, it coincides with *PRO*. Thirdly, we look at a class of rules that mix the proportionality principle with equal losses (hereafter, $LP[\alpha]$). These rules pick an α-weighted average of the proportional allocation and an allocation which equates the losses incurred by the investors. For $\alpha = 0$, the rule $LP[\alpha]$ coincides with an "unconstrained equal losses rule" (*EL*) which always equates the investors' losses. For $\alpha = 1$, it coincides with *PRO*.

For each one of these bankruptcy rules, we construct a simple game among n investors who simultaneously choose how much money to invest in a firm. The total of these investments determine the value of the firm. The firm is a lottery which either brings a positive return or goes bankrupt. In the latter case, its liquidation value is allocated among the investors according to the prespecified bankruptcy rule. For each bankruptcy rule, we analyze the Nash equilibria of the corresponding investment game. We then compare these equilibria.

In our model, agents have Constant Absolute Risk-Aversion preferences and are weakly ordered according to their degrees of risk aversion. (This ordering is without loss of generality since the agents are identical in other dimensions.) The agents do not face liquidity constraints and thus, their income levels are not relevant. However, as is standard in the literature, it is possible to interpret the agents' risk-aversion levels as a decreasing function of their income levels. (Thus, less risk averse agents can be thought of as richer, bigger investors.) Alternatively, each agent can be taken to represent an investment fund. In this case, the income level is irrelevant. The risk-aversion parameter attached to each investment fund then represents the type of that fund.

Since we do not restrict possible configurations of risk aversion, our model can be used to represent and compare societies with very different risk-aversion (or income) distributions, ranging from symmetric to asymmetric distributions with different moments. This flexibility also allows us to compare the three principles in terms of how they treat different types of agents (such as big versus small investors) as well as how they react to changes in the risk-aversion distribution.

Our analysis compares bankruptcy rules in terms of two criteria that were not considered before. Our first criterion is **total equilibrium investment** which is a simple measure of how a bankruptcy rule affects investment behavior in the economy. It is reasonable to think that a government prefers bankruptcy rules that induce higher total investment in the economy. Thus, a bankruptcy rule that induces higher total investment than *PRO* might be considered a superior alternative to it. On the other hand, it is not clear that an increase in total investment will also increase the welfare of the investors. Thus, our second criterion is **equilibrium social welfare**. Egalitarianism and utilitarianism present two competing and central notions of measuring social welfare. We therefore compare bankruptcy rules in terms of both **egalitarian** and **utilitarian** social welfare that they induce in equilibrium.

A summary of our main results is as follows. The investment game has a unique Nash equilibrium for every parameter combination and for each bankruptcy rule. These equilibria are such that, at all parameter values (i) *EL* induces higher total investment than *PRO* which in turn induces higher total investment than *EA*; (ii) *PRO* induces higher egalitarian social welfare than both *EA* and *EL* in interior equilibria; (iii) *PRO* induces higher utilitarian social welfare than *EL* in interior equilibria but its relation to *EA* depends on the parameter values (however, a numerical analysis shows that on a large part of the parameter space, *PRO* induces higher utilitarian social welfare than *EA*). Thus, in the confines of our simple model, *PRO* outperforms *EA* in almost every criterion. Also, switching from *PRO* to *EL* increases total investment but decreases both egalitarian and utilitarian social welfare.

[2] This is not surprising considering that in US between 1999 and 2009, more than 551 000 firms filed for Chapter 7 bankruptcy and more than 22.16 billion USD were allocated in these cases (see http://www.justice.gov/ust/index.htm).

[3] Procedures on the liquidation of the firm and its allocation among creditors exist in bankruptcy laws of every country. For examples, see Chapter 7 of the US Bankruptcy Code or the Receivership code in UK. In some countries such as Sweden or Finland, these procedures provide the only option for the resolution of bankruptcy. Bankruptcy laws of some other countries, such as US, also offer procedures (such as Chapter 11) for reorganization of the bankrupt firm.

[4] This is a very old and common practice, referred to as a *pari passu* distribution; the term meaning "proportionally, at an equal pace, without preference" (see Black's Law Dictionary, 2004).