Available online at www.sciencedirect.com

SciVerse ScienceDirect

Systems Engineering Procedia 5 (2012) 150 – 155

Procedia
Systems Engineering

International Symposium on Engineering Emergency Management 2011

# Logarithm Utility Maximization Portfolio Engineering with Bankruptcy Control: a Nonparametric Estimation Framework

Qing-hua Ma, Hai-xiang Yao *, Shi-Yang Li

*School of Informatics, Guangdong University of Foreign Studies, Guangzhou 510006, China*

**Abstract**

Under the assumption that investors have the logarithm utility function, this paper adopts the methodology of nonparametric estimation and the expected utility maximization (EUM) model to explore a portfolio engineering problem with bankruptcy control. First, we obtain the nonparametric estimated calculation formula for expected utility by using the nonparametric estimation. Then, sequential quadratic programming (SQP) algorithm for the optimal investment strategy of the EUM model is given. Finally, a numerical portfolio engineering example based on real data of Chinese stock market is presented.



*Keywords:* Portfolio engineering; Bankruptcy control; Logarithm utility maximization; Nonparametric estimation;

## 1. Introduction

Expected utility maximization (EUM) model initiated by [1] and mean-variance model established by [2] are the two principal areas for studying the portfolio selection (engineering) problem in portfolio engineering. Using the EUM model, [3] and [4] have study the optimal investment-consumption strategies problem in discrete time and continuous time, respectively. On the other hand, [5] and [6] expanded the mean-variance model to cases of discrete time and continuous time, respectively.

However, in portfolio engineering, investors would often be confronted with the risk of bankruptcy when they use directly the investment strategies of EUM model or mean-variance model. For this reason, using discrete time mean-variance model, [7] and [8] investigated the portfolio engineering problem with bankruptcy control. [9] solved a continuous-time mean-variance portfolio selection problem with bankruptcy prohibition. On the other hand, logarithm utility function is always the frequently-used utility function in the EUM model. One reason is that it is easy to compute optimal strategies explicitly, see, for example, [10] and [11]. The other reason is that the logarithm optimal strategy also maximizes the long term growth rate in an almost-sure sense (see [12]), that is the goal for most investor to pursue. [13] studied the optimal long term growth rate of wealth. However, most of the above-mentioned literatures are under the assumption that return on assets obey some specific probability distribution type, such as normal distribution or geometry Brownian movement. But it is well known that the finance market is complex and changeable. It is very difficult for us to know the probability distribution type of asset return. On the other hand, nonparametric estimation method need not make any hypothesis about the distribution type, and the calculated results are completely driven by market sample data (see [14]), which can makes it adaptable in the context of capricious financial market. So in recent years, some scholars began to adopt nonparametric estimation

 doi:10.1016/j.sepro.2012.04.024

method to study the calculation of financial risk; we refer [15-16] to the readers. But to the best knowledge of the authors, there is no literature to consider the EUM problem by using nonparametric estimation method. Basing on logarithm EUM model and using nonparametric estimation method, we will investigate a portfolio engineering (selection) problem with risk control over bankruptcy in this paper.

This paper is organized as follows. In section two, a EUM problem with risk control over bankruptcy is established. In section three, we obtain the estimated calculation formula of expected utility by using nonparametric estimation of the portfolio return's density function. In section four, we give the sequential quadratic programming (SQP) algorithm for obtaining the optimal investment strategy. Finally, we indicate a portfolio engineering numerical example basing on real data of Chinese stock market to show the validity and the practicability of our results.

## 2. EUM model with risk control over bankruptcy

Suppose that there are $n$ assets with return vector $\vec{\xi} = (\xi_1, \xi_2, \cdots \xi_n)'$ for investors to invest. Let $W = (w_1, w_2, \cdots, w_n)'$ denote the portfolio of the assets. Here $A'$ denotes the transpose of matrix $A$. Then, return of portfolio is $\xi_p := \sum_i^n w_i \xi_i = W'\vec{\xi}$. In reality, there is one another problem for us to consider, namely to avoid bankruptcy. Following [7] and [8], a bankruptcy occurs when the wealth or portfolio return of the investor falls below a predefined ''disaster'' level $b$. Therefore, in the process of investment, we should control the probability $P(\xi_p < b)$ of bankruptcy. According to Tchebycheff inequality, we have $P(\xi_p < b) \le \frac{\mathrm{Var}[\xi_p]}{(\mathrm{E}[\xi_p] - b)^2}$. Then, controlling the risk of bankruptcy can be achieved by setting a small value $d$ such that $\frac{\mathrm{Var}[\xi_p]}{(\mathrm{E}[\xi_p] - b)^2} \le d$, i.e. $\mathrm{Var}[\xi_p] \le d(\mathrm{E}[\xi_p] - b)^2$. Therefore, EUM portfolio selection model with a bankruptcy control can be represented as the following optimization problem

$$\begin{cases} \max \ \mathrm{E}[U(W'\vec{\xi})] \\ s.t. \quad W'\vec{1} = 1, \ \mathrm{Var}[\xi_p] \le d(\mathrm{E}[\xi_p] - b)^2, \end{cases} \tag{1}$$

where $\vec{1}$ denotes a column vector whose $n$ entries are all ones, $U(\cdot)$ is the utility function corresponding to investor. In order to guarantee the optimal solution exist, $U(\cdot)$ often needs to satisfy some mathematical properties, such as concavity and monotonic increase. We also suppose that $U(\cdot)$ is logarithm function, i.e. $U(x) = \ln x$.

## 3. EUM model base on nonparametric estimation framework

In the paper, we will obtain the estimated formula of expected utility by adopting the nonparametric method to estimate the distribution of $\xi_p$ or $\vec{\xi}$, and make further efforts to investigate the EUM portfolio selection problem. Since the asset's returns vector $\vec{\xi}$ is a multidimensional random vector, if we adopt nonparametric method to estimate its density function, the convergence rate would be very slow, which sometimes referred to as the "curse of dimensionality" (see [14]). For the sake of overcoming the problem of "curse of dimensionality", we will obtain the nonparametric estimation for $\mathrm{E}[U(W'\vec{\xi})]$ by estimating the density of return of portfolio, that is only one dimension.

Now we introduce some preliminary knowledge (details see in [14]). Nonparametric estimation of probability density function (PDF) $p(x)$ of univariate random variable $X$ with sample set $\{ X_1, X_2, \cdots, X_T \}$ is

$$\hat{p}(x) = T^{-1} h^{-1} \sum_{i=1}^{T} k(\frac{X_i - x}{h}), \tag{2}$$